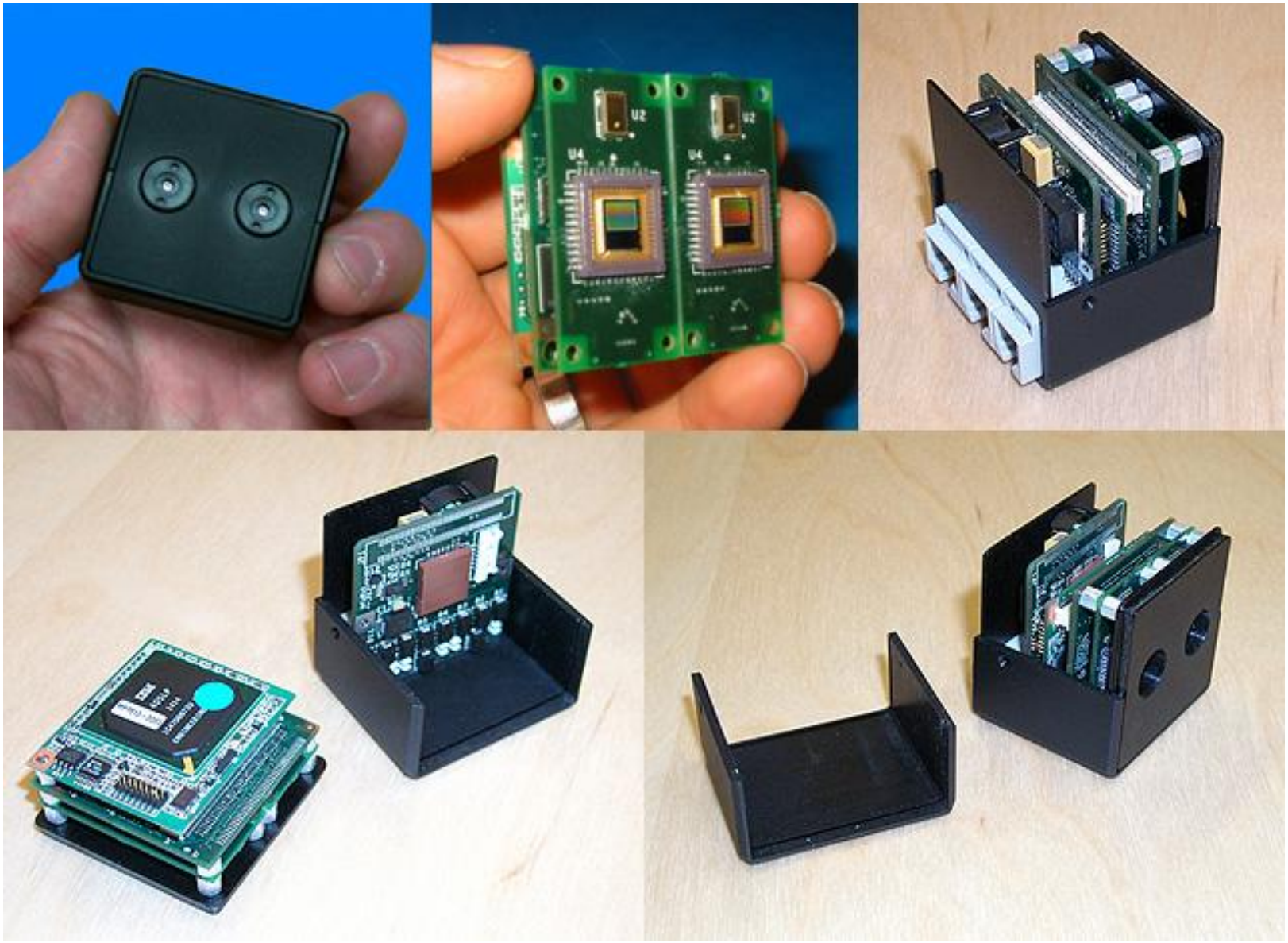
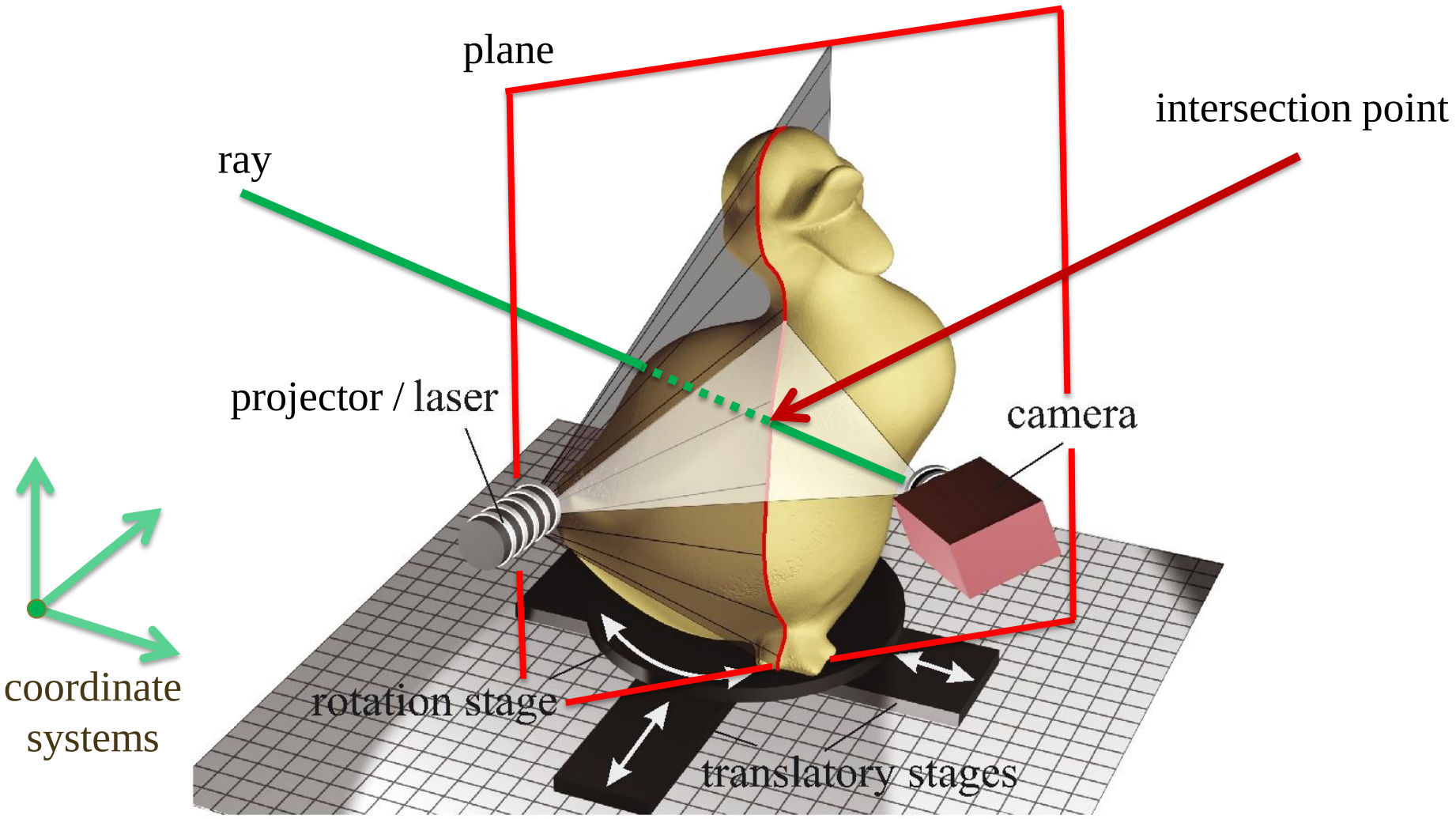
The Mathematics of 3D Triangulation

What is a Camera ?

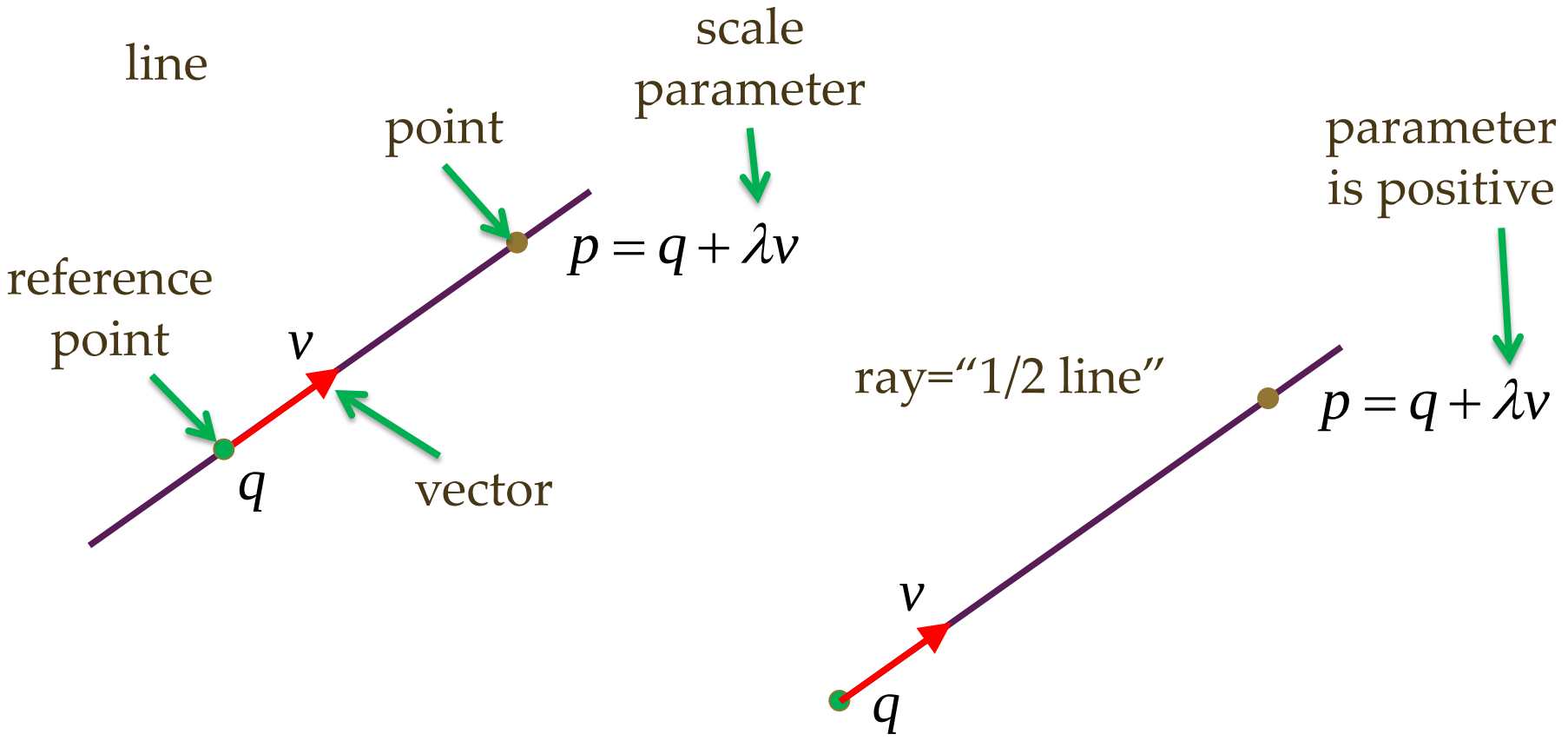


What is a Camera ?

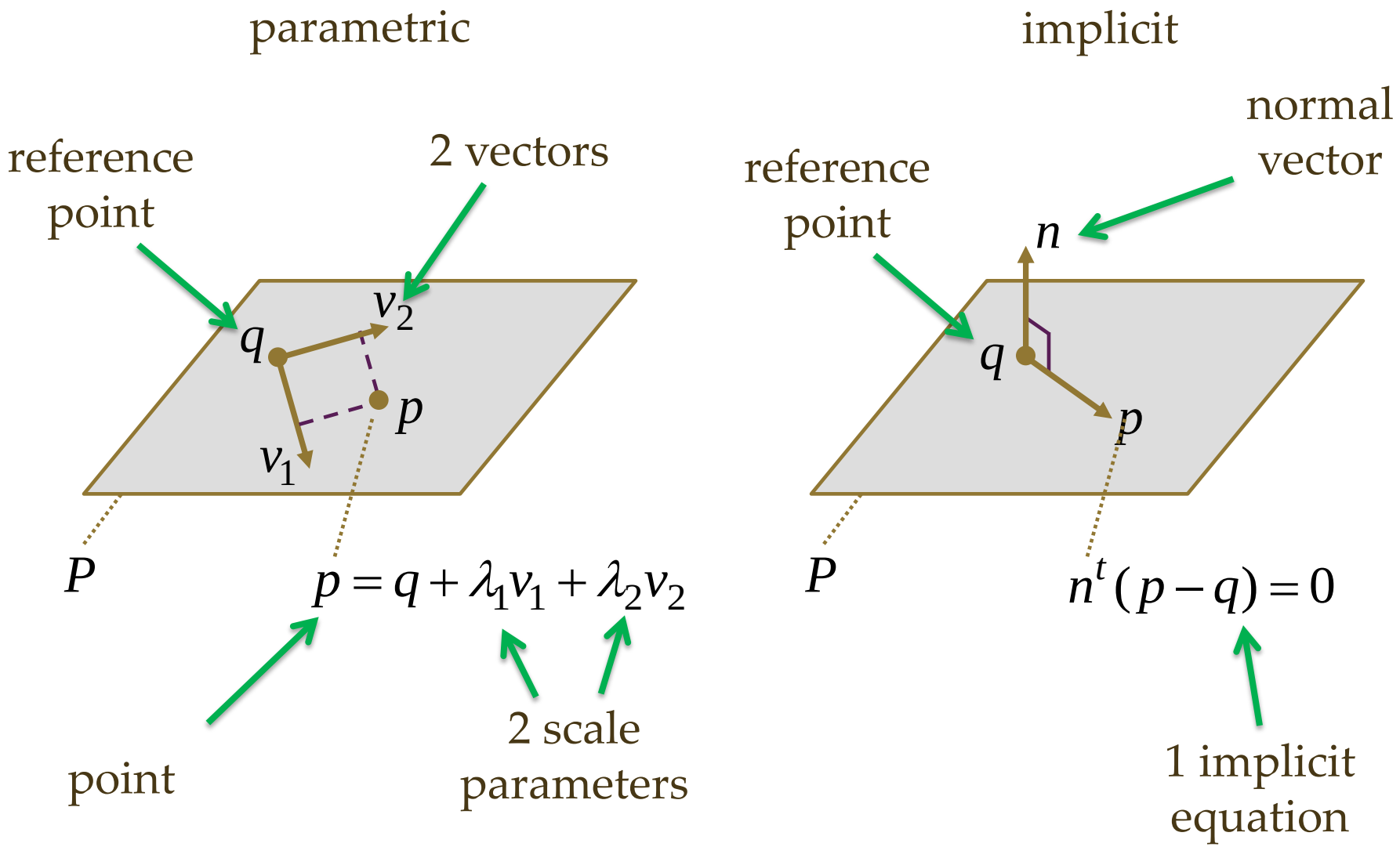
3D Triangulation: Ray-Plane Intersection



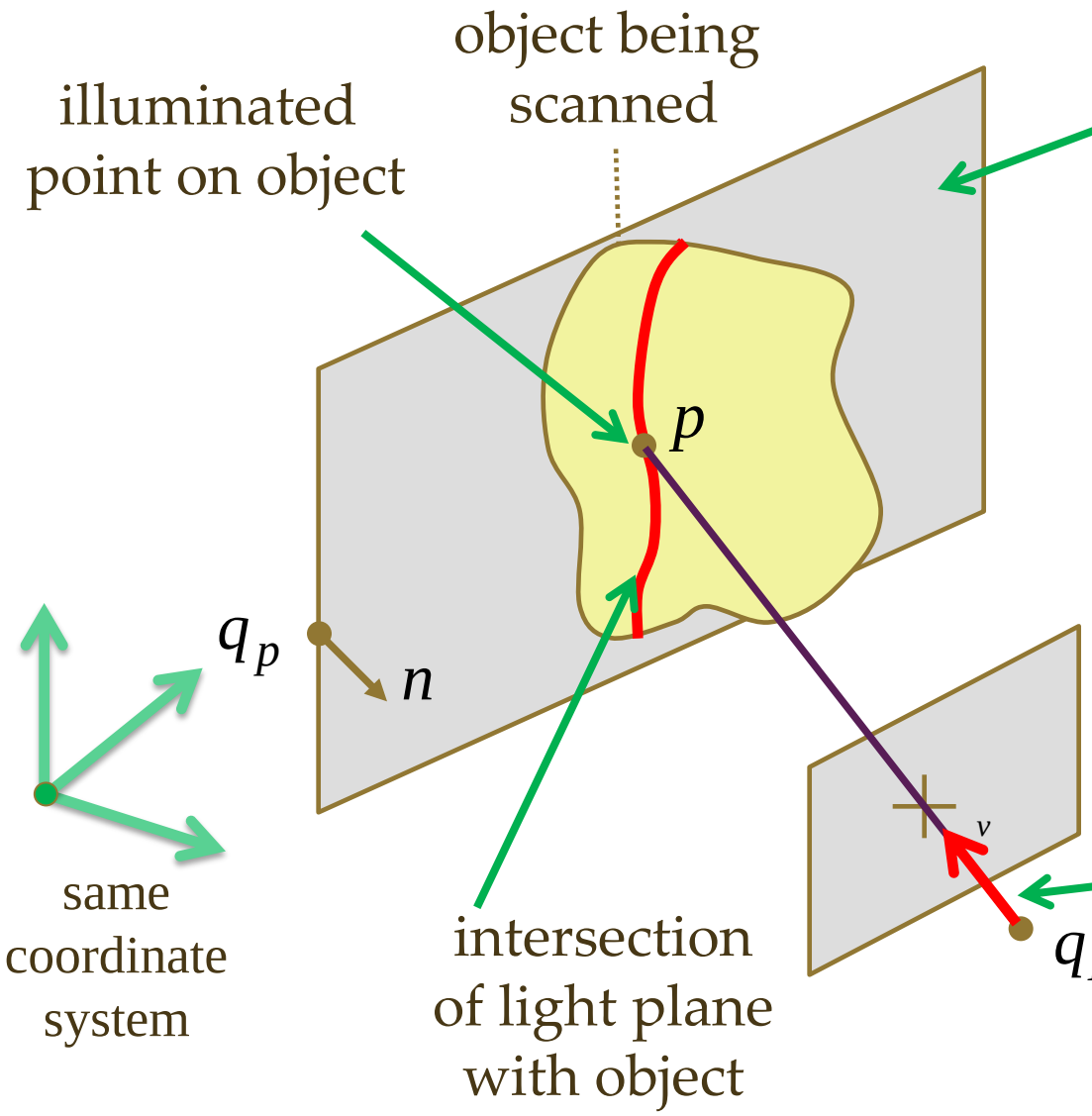
Representation of Lines and Rays



Representation of Planes



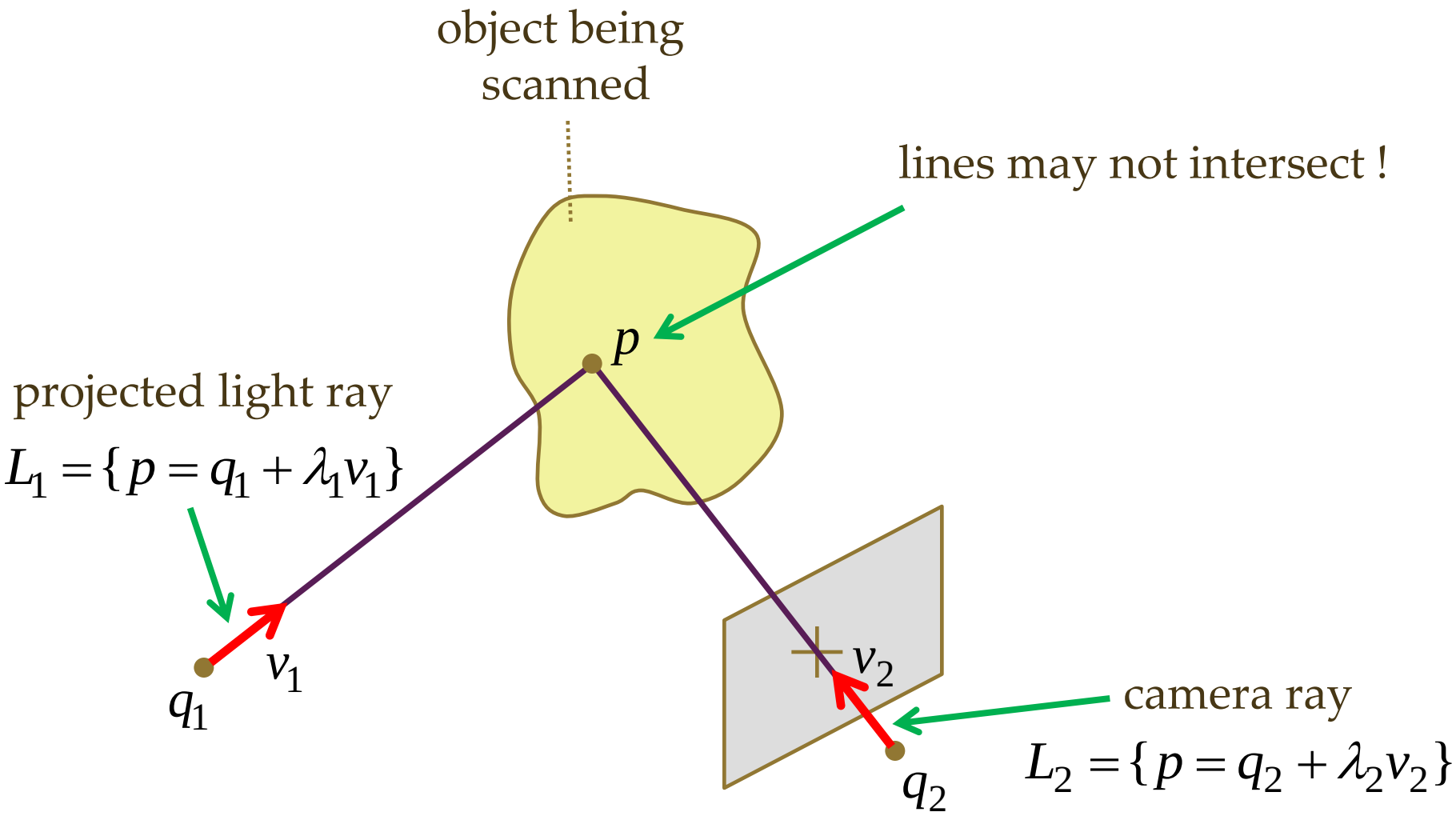
Triangulation by Line-Plane Intersection



$$n^t(\lambda v + q_L - q_p) = 0$$

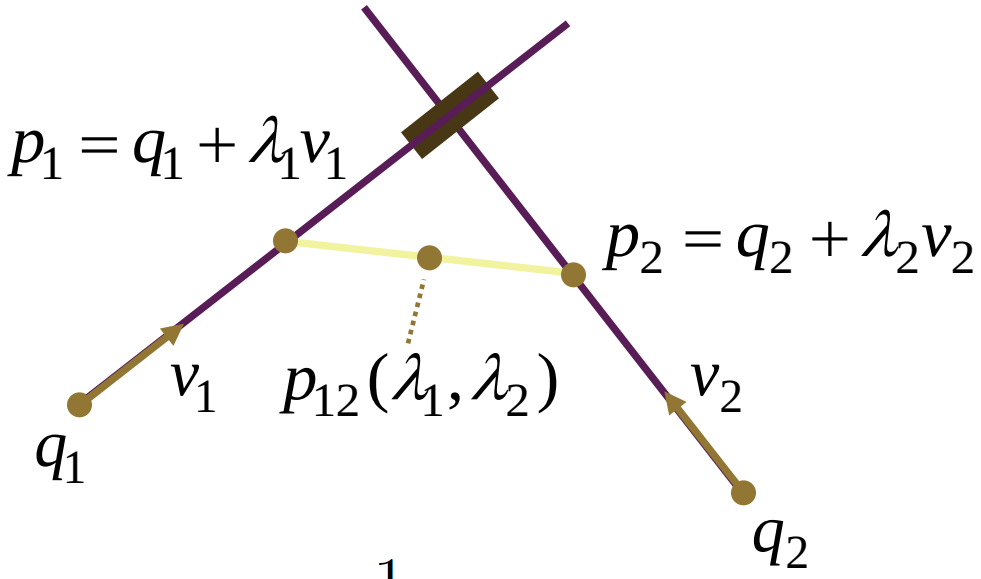
$$\lambda = \frac{n^t(q_P - q_L)}{n^t v}$$

Triangulation by Line-Line Intersection



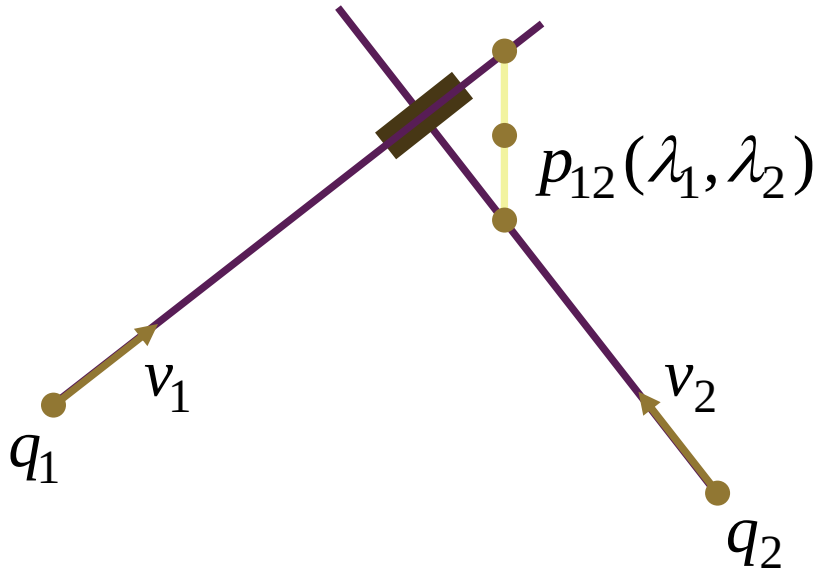
Approximate Line-Line Intersection

Midpoint of segment joining arbitrary points in the two lines



$$p_{12} = p_1 + \frac{1}{2}(p_2 - p_1)$$

Least-squares approach



Find parameters which minimize

$$\|(q_2 + \lambda_2 v_2) - (q_1 + \lambda_1 v_1)\|^2$$

Approximate Line-Line Intersection

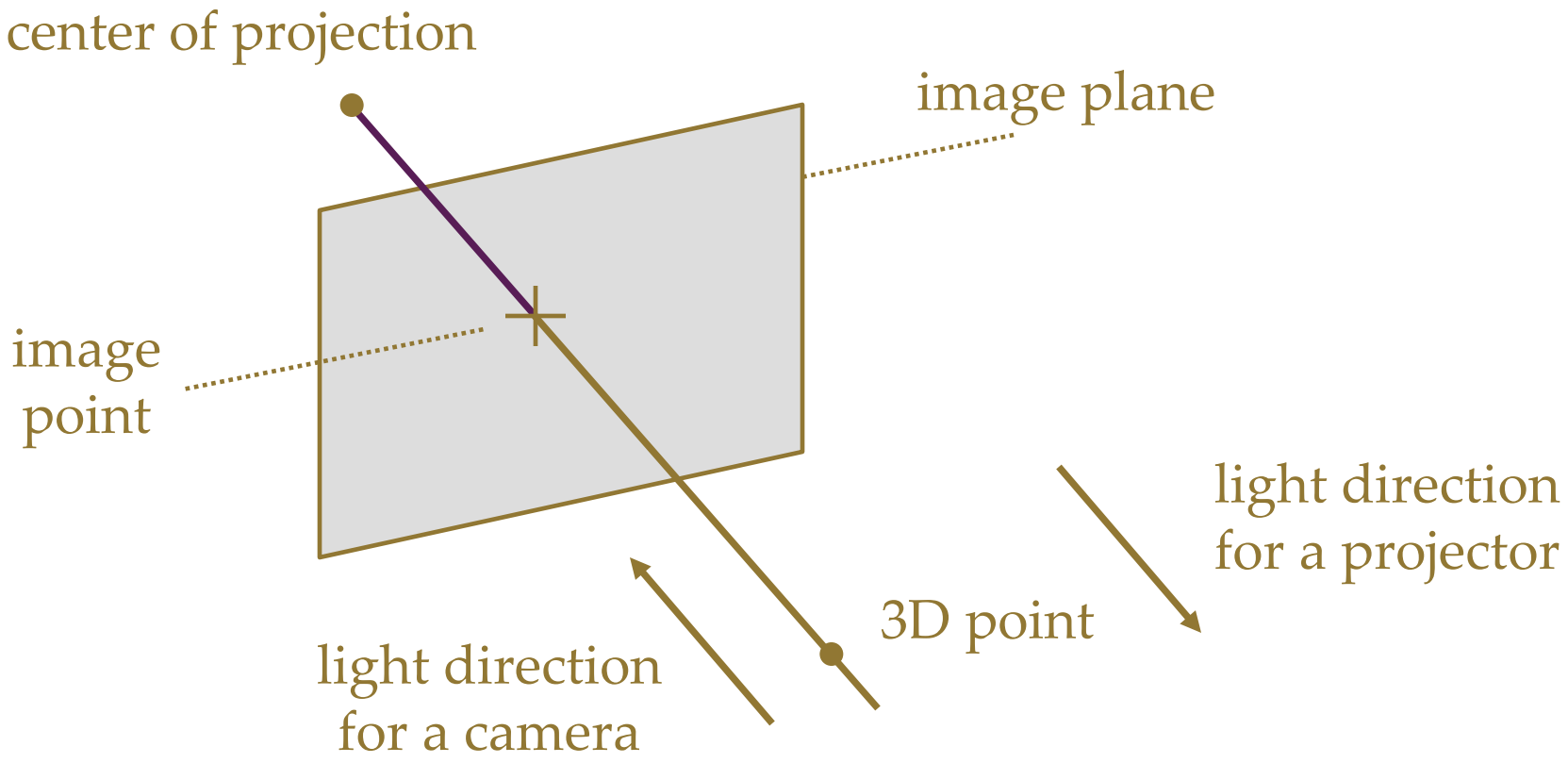
$$p_{12} = p_1 + \frac{1}{2}(p_2 - p_1)$$

$$p_1 = q_1 + \lambda_1 v_1$$

$$p_2 = q_2 + \lambda_2 v_2$$

$$\begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} \|v_1\|^2 & -v_1^t v_2 \\ -v_2^t v_1 & \|v_2\|^2 \end{pmatrix}^{-1} \begin{pmatrix} v_1^t (q_2 - q_1) \\ v_2^t (q_1 - q_2) \end{pmatrix}$$

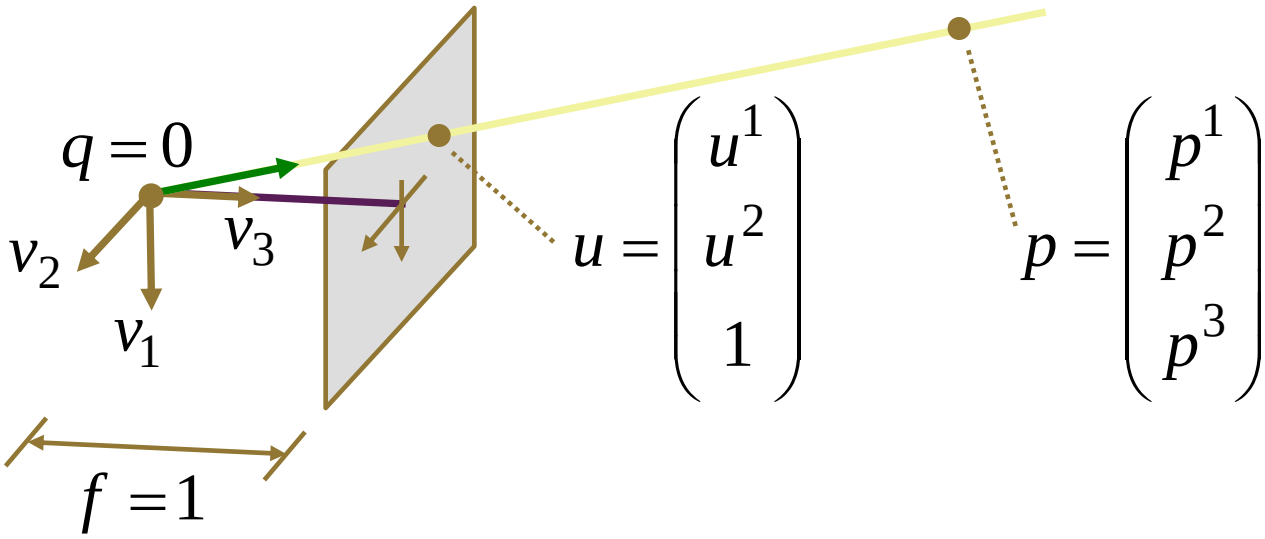
Perspective Projection (Pinhole Model)



Calibration: mapping from image points to rays

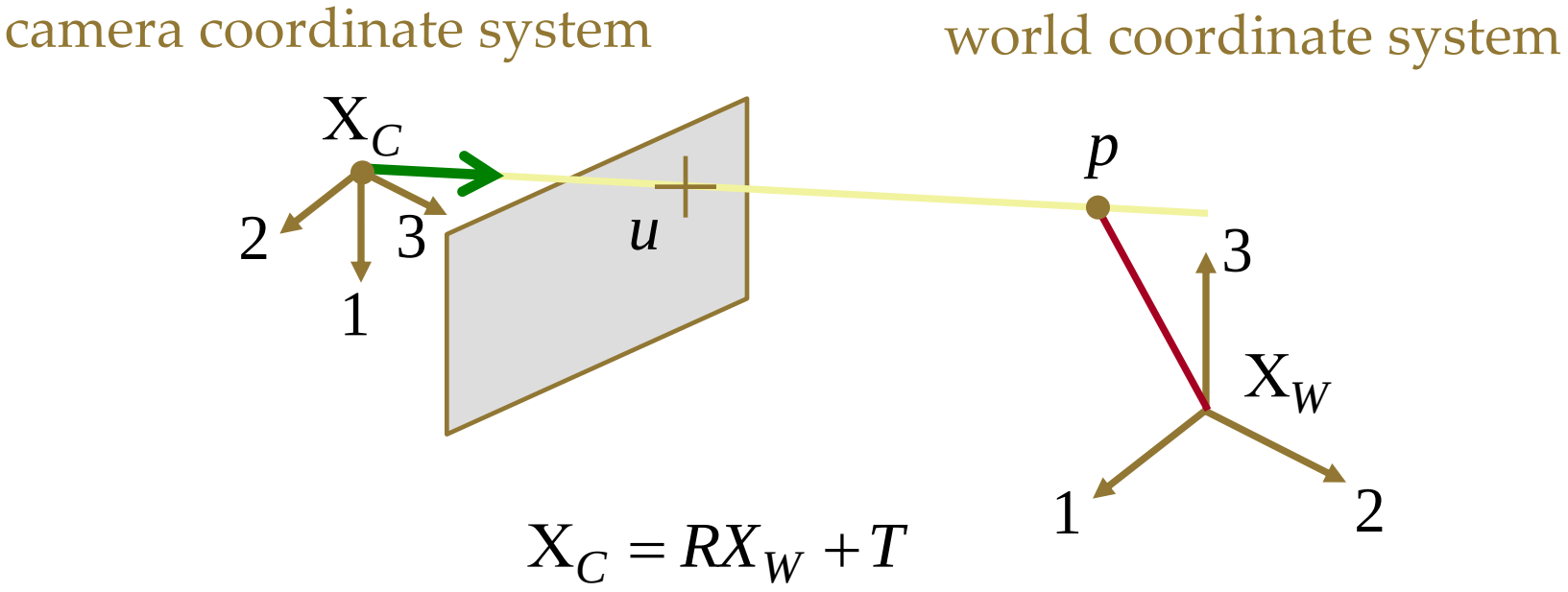
The Ideal Pinhole Camera

camera coordinate system = world coordinate system



$$\begin{pmatrix} p^1 \\ p^2 \\ p^3 \end{pmatrix} = \lambda \; [v_1|v_2|q] \begin{pmatrix} u^1 \\ u^2 \\ 1 \end{pmatrix} \qquad [v_1|v_2|q] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The General Pinhole Model



$$\lambda u = R p_W + T$$

extrinsic parameters

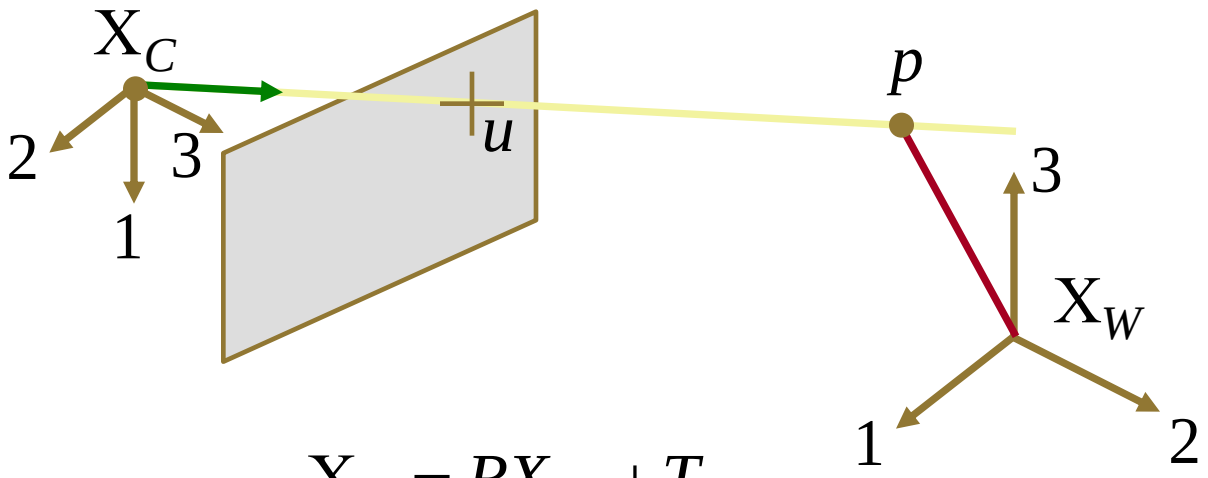
Ideal assumptions

- Image lengths = world lengths
- Focal length = 1
- Image origin = optical center
- Image plane spanned by two basis vectors

The General Pinhole Model

camera coordinate system

world coordinate system



$$X_C = RX_W + T$$

intrinsic parameters

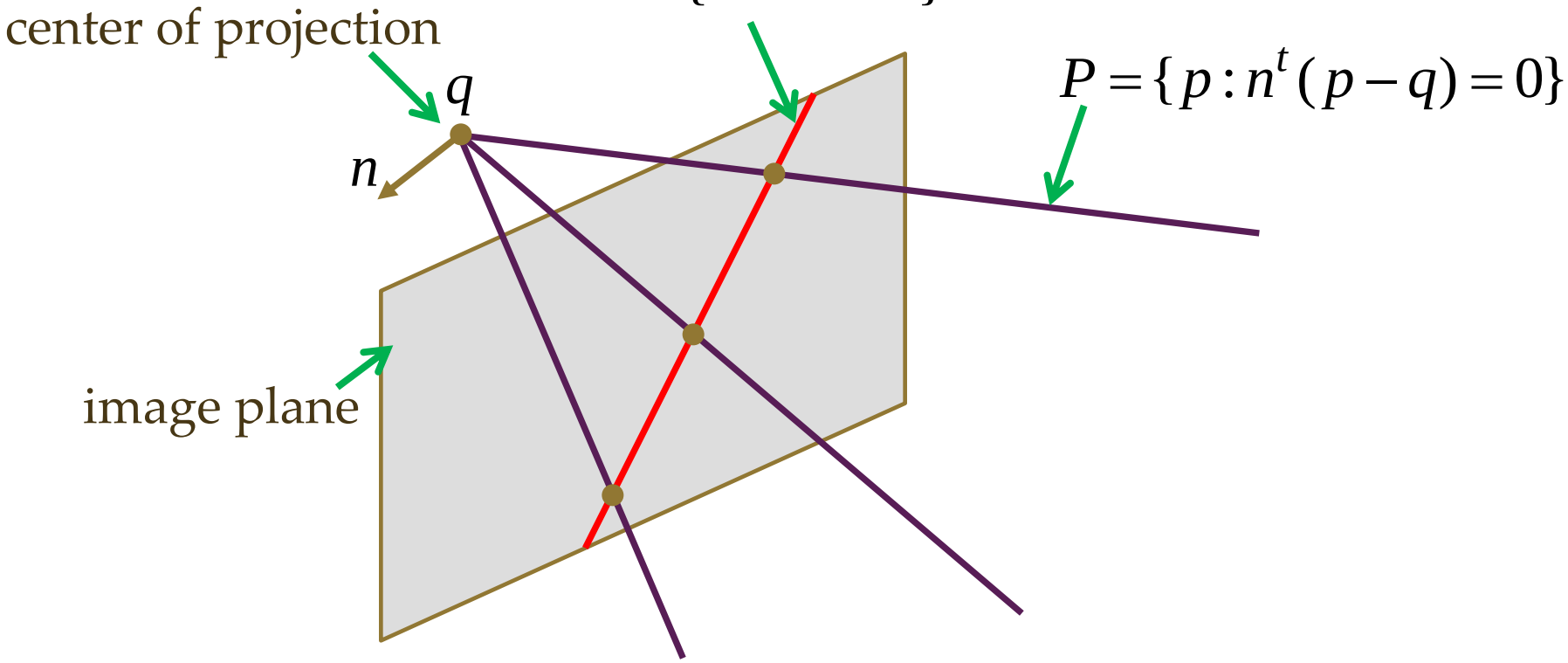
$$\lambda u = K(R p_W + T)$$

$$K = \begin{pmatrix} f s_1 & f s_\theta & o^1 \\ 0 & f s_2 & o^2 \\ 0 & 0 & 1 \end{pmatrix}$$

Plane Defined by Image Line and Projection Center

Implicit equation of line in image coordinates

$$L = \{u : l^t u = 0\}$$



$$0 = \lambda l^t u = l^t (R\, p_W \; + \; T) = (R^t l)^t \, (p_W - (-R^t T)) \; .$$