Introducción a la Fotografia 3D

UBA/FCEN Marzo 27 – Abril 12 2013

Clase 2 : Miercoles Abril 3

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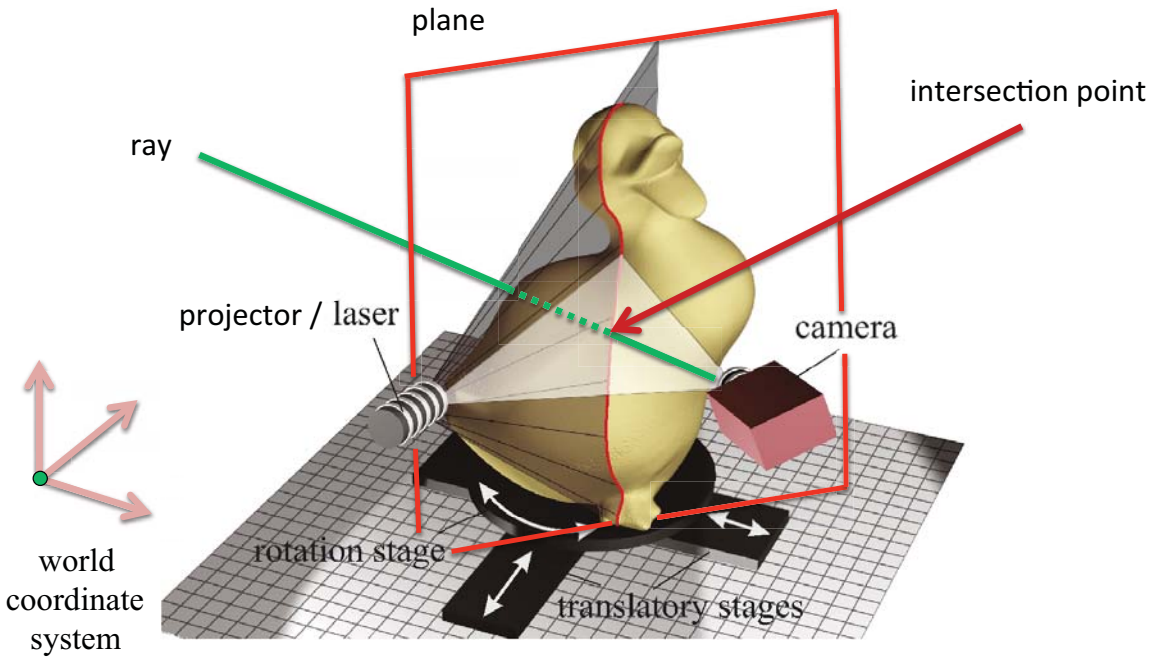
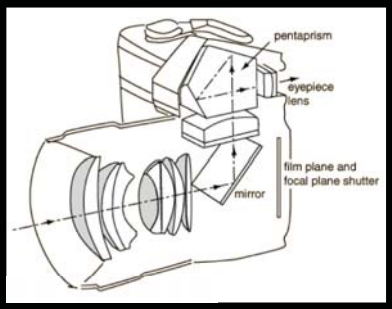
Las matemáticas de la triangulacion 3D

- Triangulation por interseccion de linea y plano
- Triangulation por interseccion de linea y linea
- Puntos, vectores, lineas, rayos, and planos
- Que es una camara?
- Que es una image?
- …

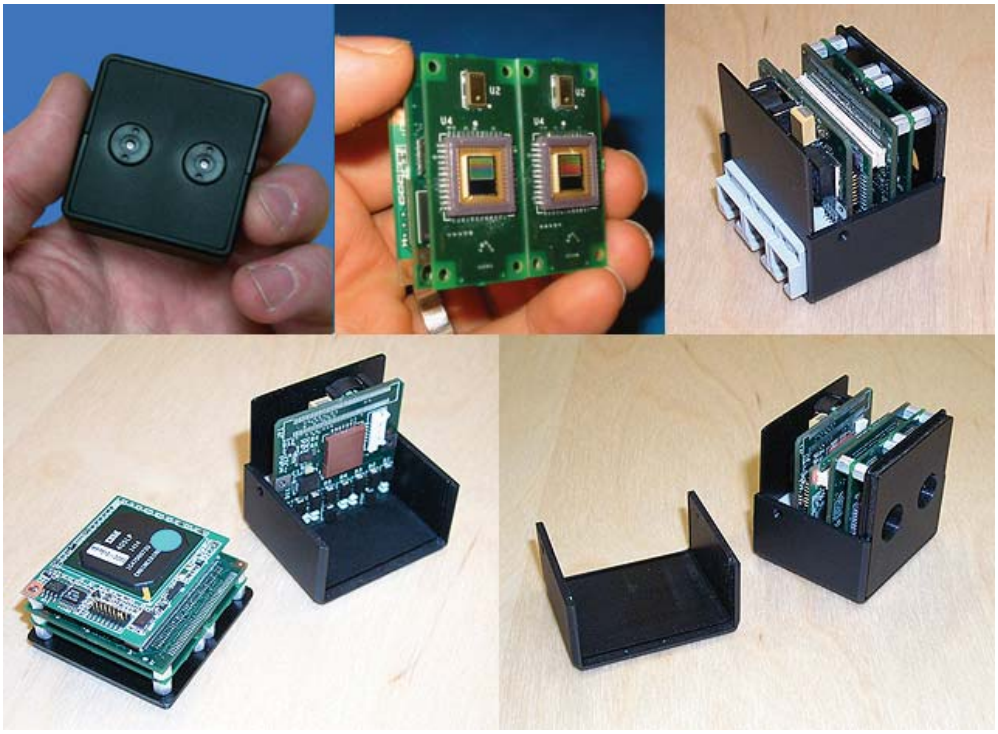
3D triangulation: ray-plane Intersection



Que es una Camara?



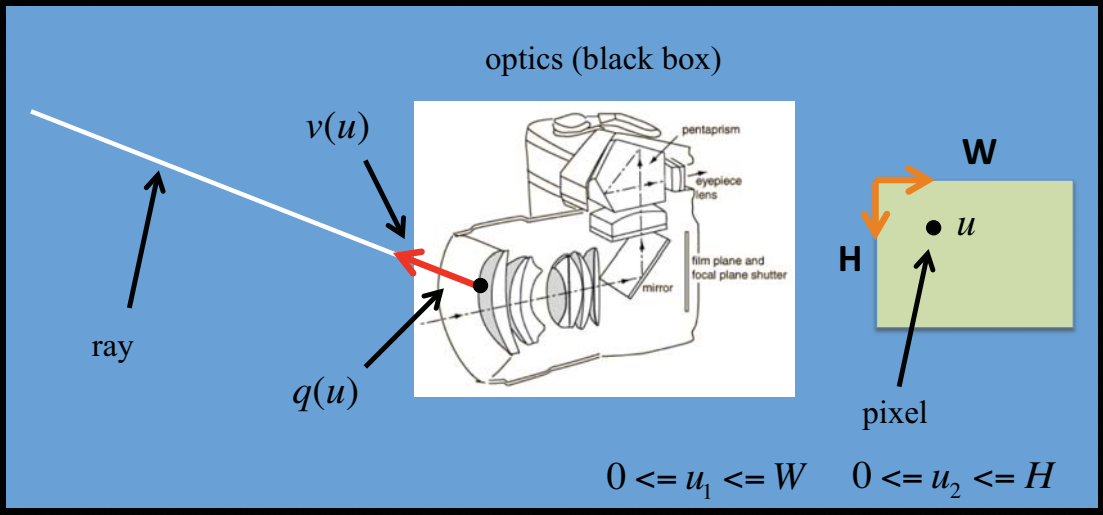
What is a Camera ?



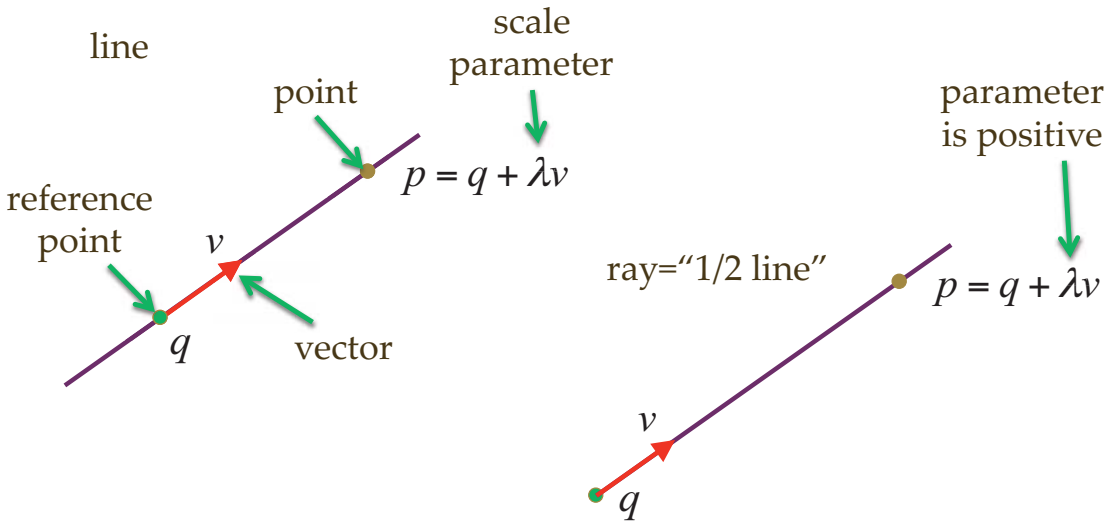
What is a Camera ?

Modelo Geometrico General de una Camara

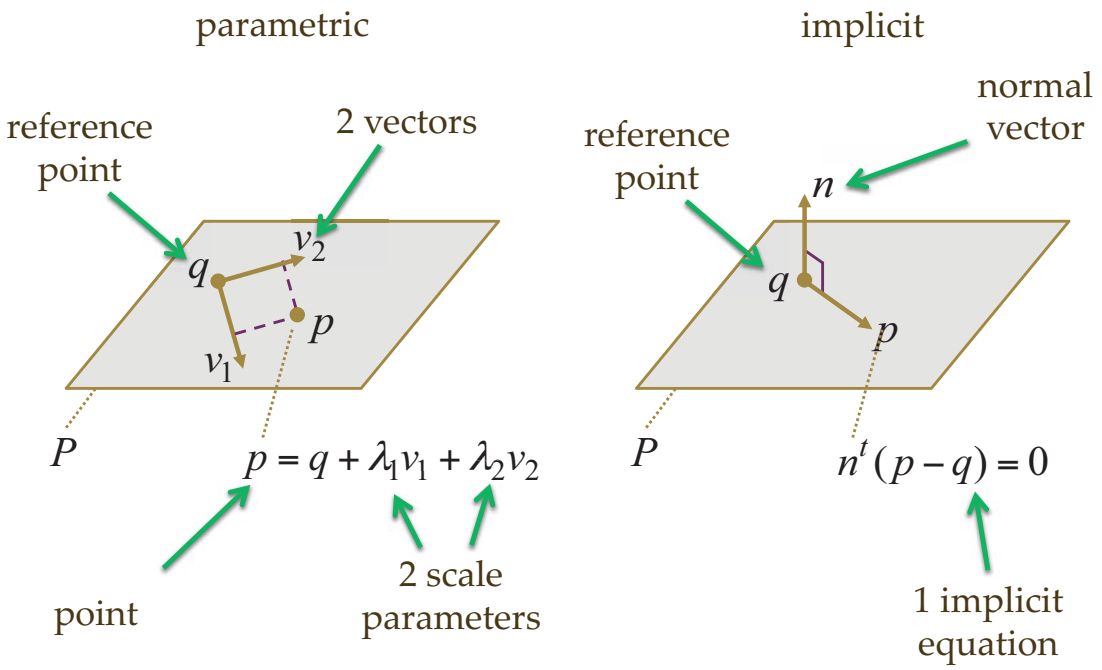
- Funcion que a cada punto de la imagen le hace corresponder un rayo
- El dominio esta contenido en un rectangulo y la funcion es continua
- En muchos casos el análisis es mas simple en el espacio de los rayos



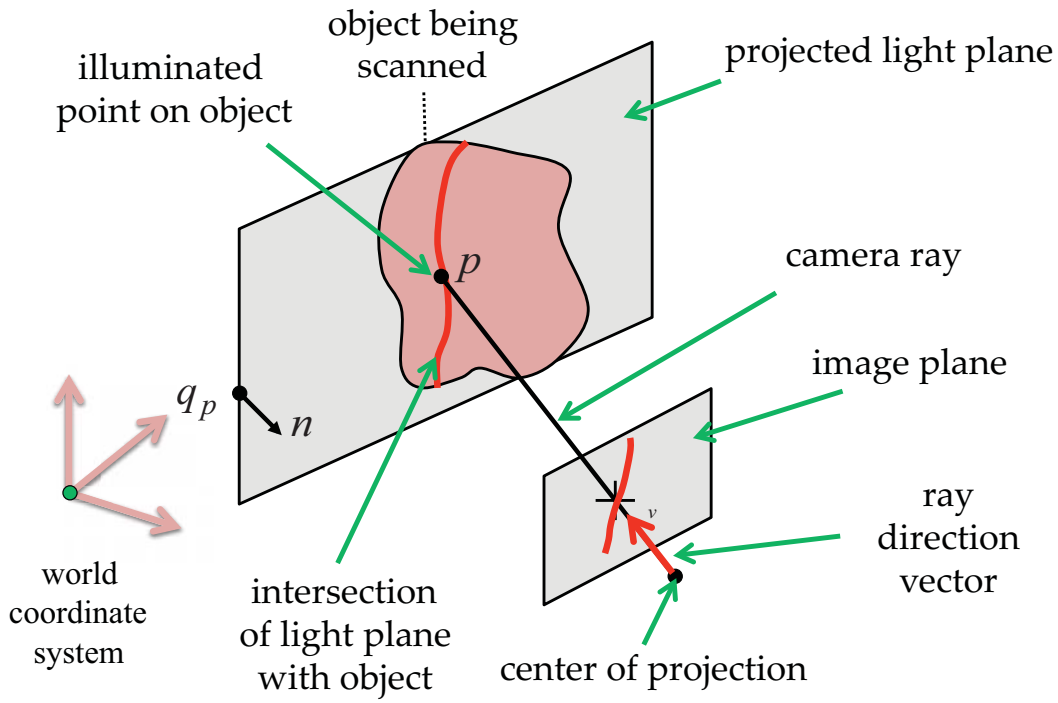
Representation of Lines and Rays



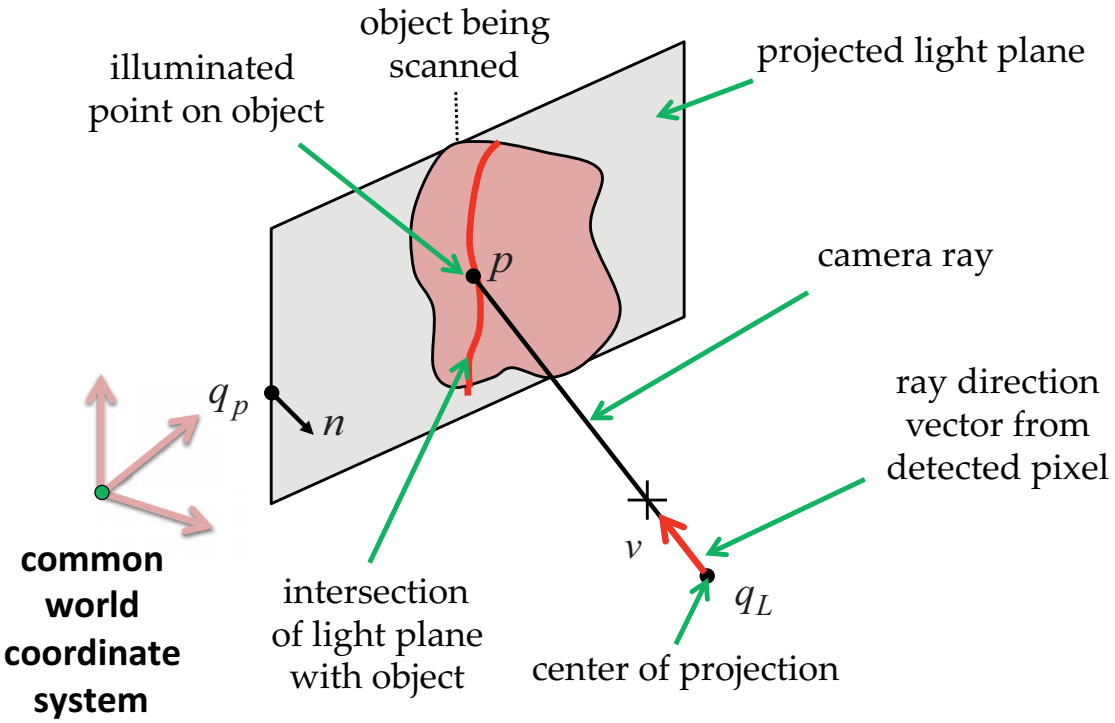
Representation of Planes



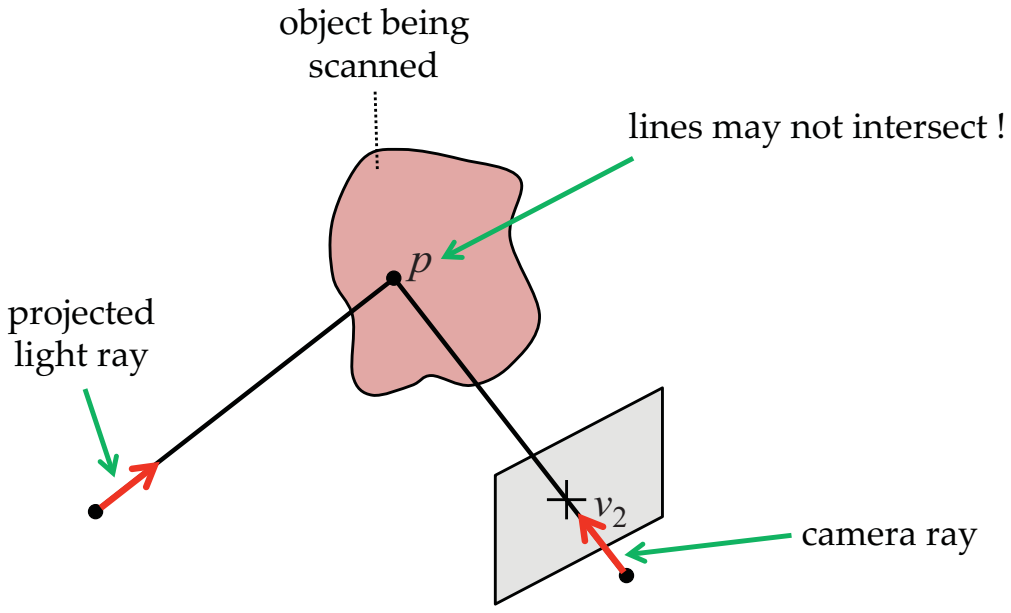
3D Triangulation by Line-Plane intersection



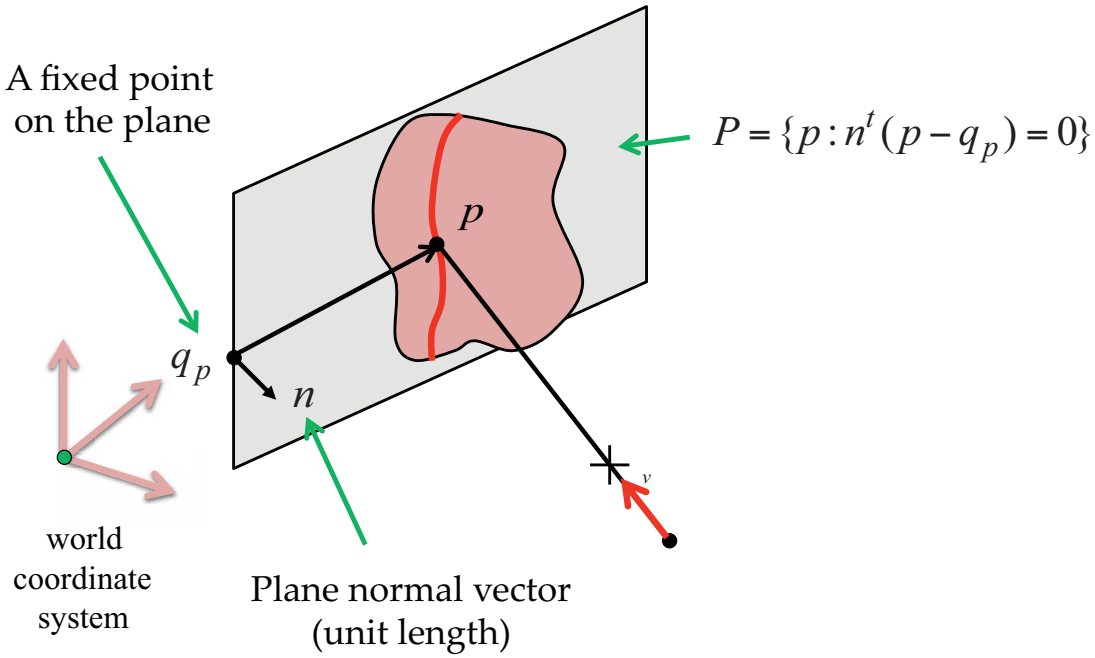
If camera and projector are calibrated



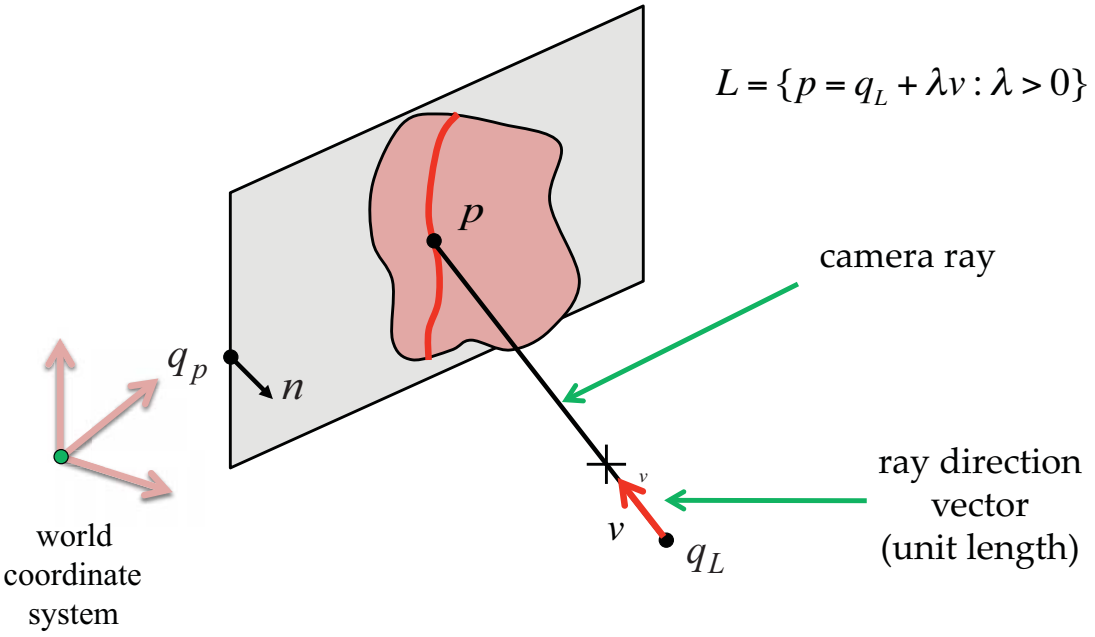
3D Triangulation by Line-Line Intersection



Implicit equation of the plane

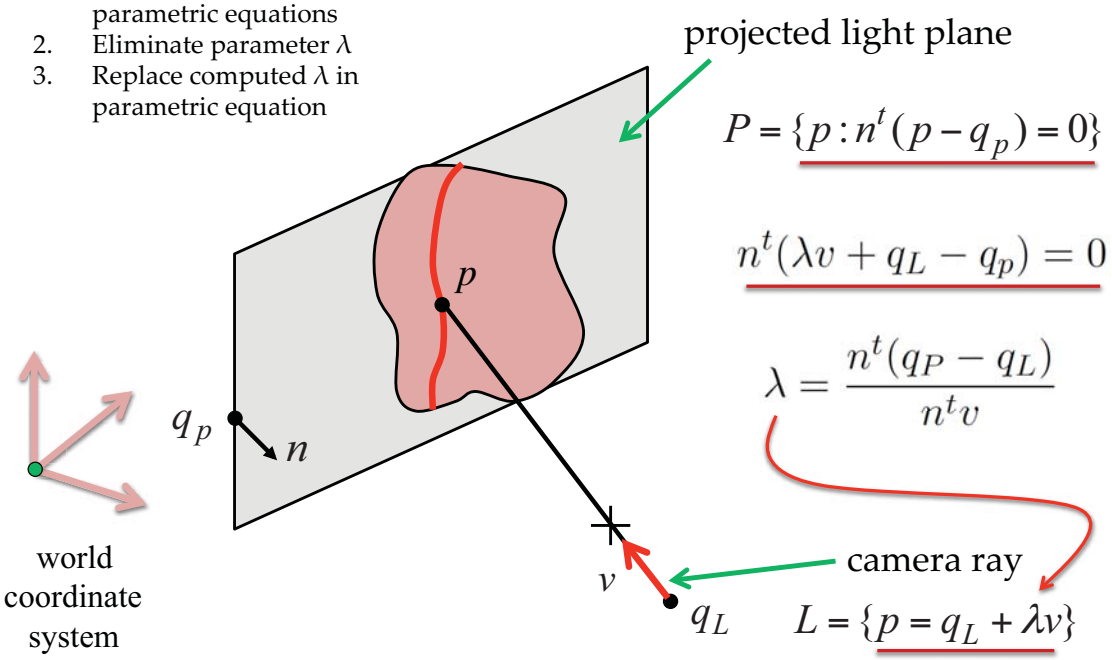


Parametric equation of the ray

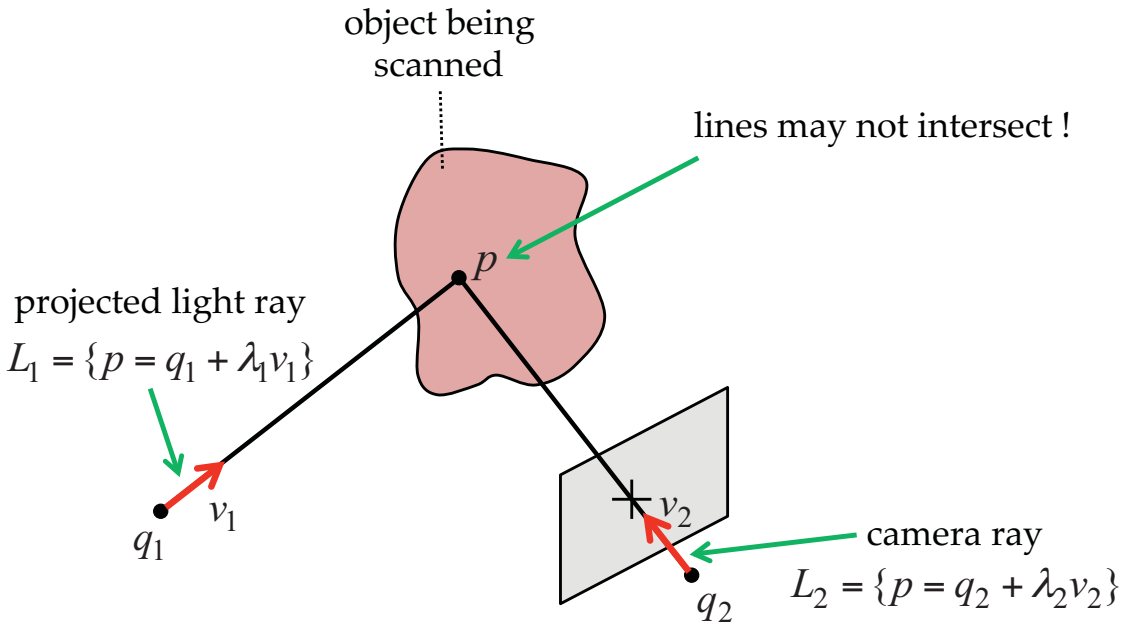


Triangulation by Line-Plane Intersection

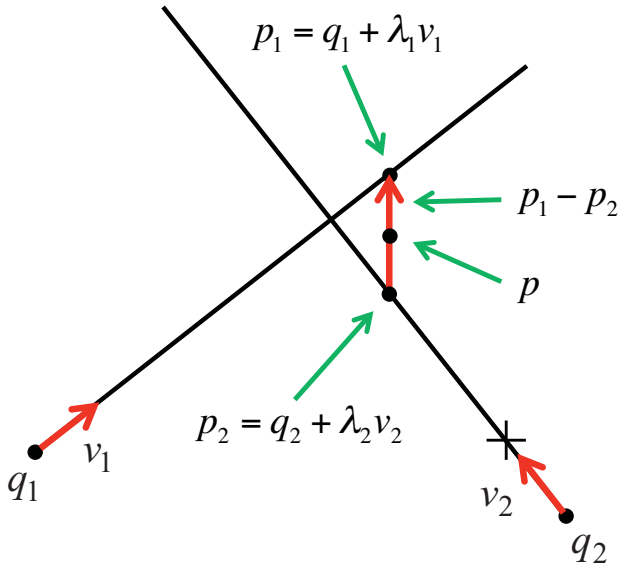
1. Compose implicit and parametric equations
2. Eliminate parameter λ
3. Replace computed λ in parametric equation



Triangulation by Line-Line Intersection



Triangulation by Line-Line Intersection



$$L_1 = \{p_1 = q_1 + \lambda_1 v_1\}$$
$$L_2 = \{p_2 = q_2 + \lambda_2 v_2\}$$

Minimize

$$E(\lambda_1, \lambda_2) = dist(p_2 - p_1)^2$$

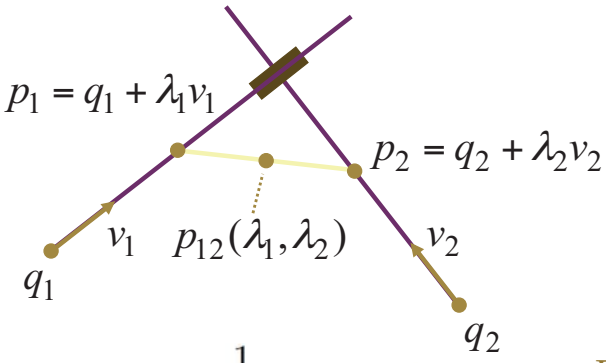
Necessary conditions

$$v_1^t (p_1 - p_2) = 0$$
$$v_2^t (p_2 - p_1) = 0$$

$$p = (p_1 + p_2)/2$$

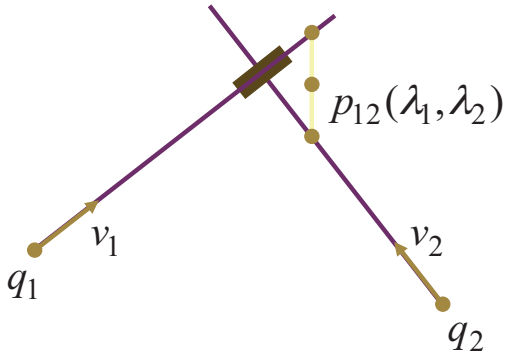
Approximate Line-Line Intersection

Midpoint of segment joining arbitrary points in the two lines



$$p_{12} = p_1 + \frac{1}{2}(p_2 - p_1)$$

Least-squares approach



Find parameters which minimize

$$\|(q_2 + \lambda_2 v_2) - (q_1 + \lambda_1 v_1)\|^2$$

Approximate Line-Line Intersection

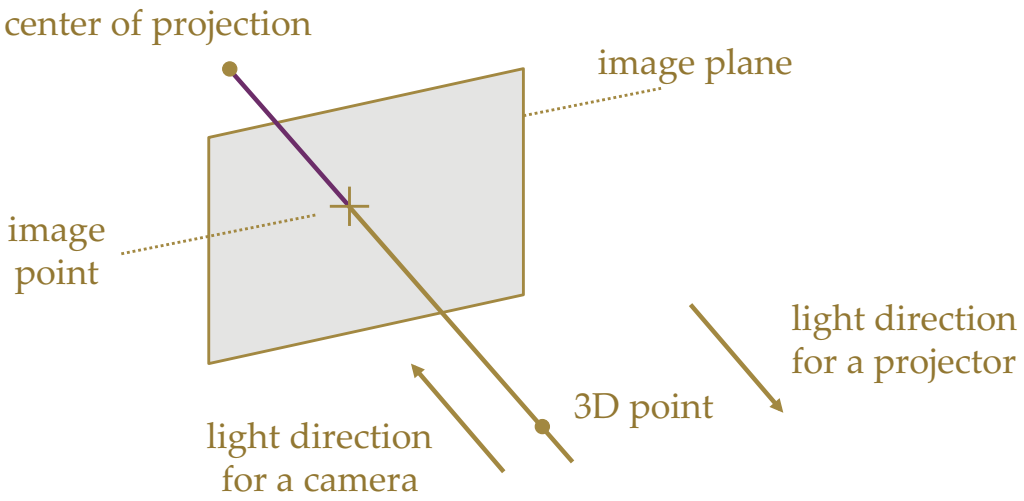
$$p_{12} = p_1 + \frac{1}{2}(p_2 - p_1)$$

$$p_1 = q_1 + \lambda_1 v_1$$

$$p_2 = q_2 + \lambda_2 v_2$$

$$\begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} \|v_1\|^2 & -v_1^t v_2 \\ -v_2^t v_1 & \|v_2\|^2 \end{pmatrix}^{-1} \begin{pmatrix} v_1^t (q_2 - q_1) \\ v_2^t (q_1 - q_2) \end{pmatrix}$$

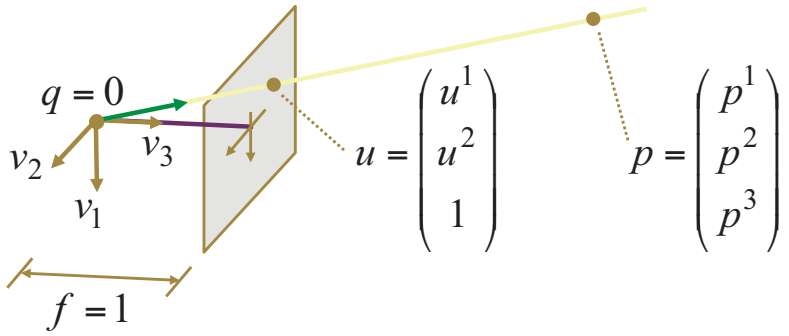
Perspective Projection (Pinhole Model)



Calibration: mapping from image points to rays

The Ideal Pinhole Camera

camera coordinate system = world coordinate system

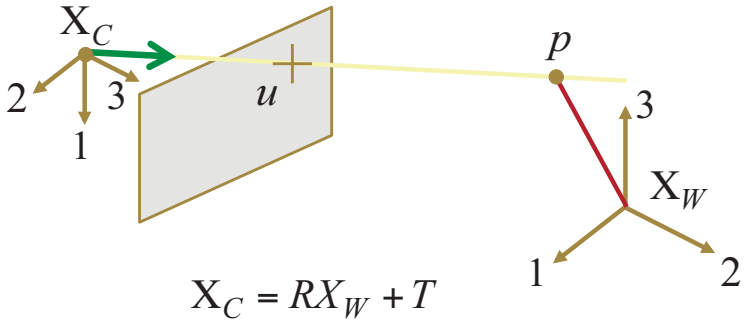


$$\begin{pmatrix} p^1 \\ p^2 \\ p^3 \end{pmatrix} = \lambda \; [v_1|v_2|q] \begin{pmatrix} u^1 \\ u^2 \\ 1 \end{pmatrix} \qquad [v_1|v_2|q] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The General Pinhole Model

camera coordinate system

world coordinate system



$$X_C = RX_W + T$$

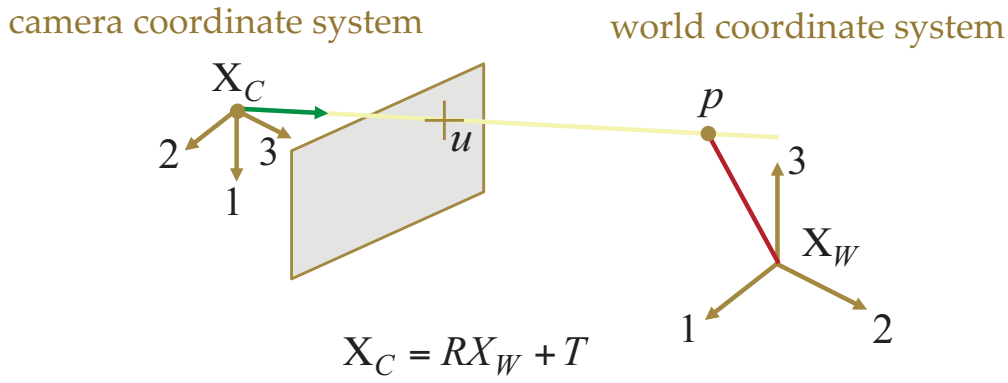
$$\lambda u = R p_W + T$$

extrinsic parameters

Ideal assumptions

- Image lengths = world lengths
- Focal length = 1
- Image origin = optical center
- Image plane spanned by two basis vectors

The General Pinhole Model



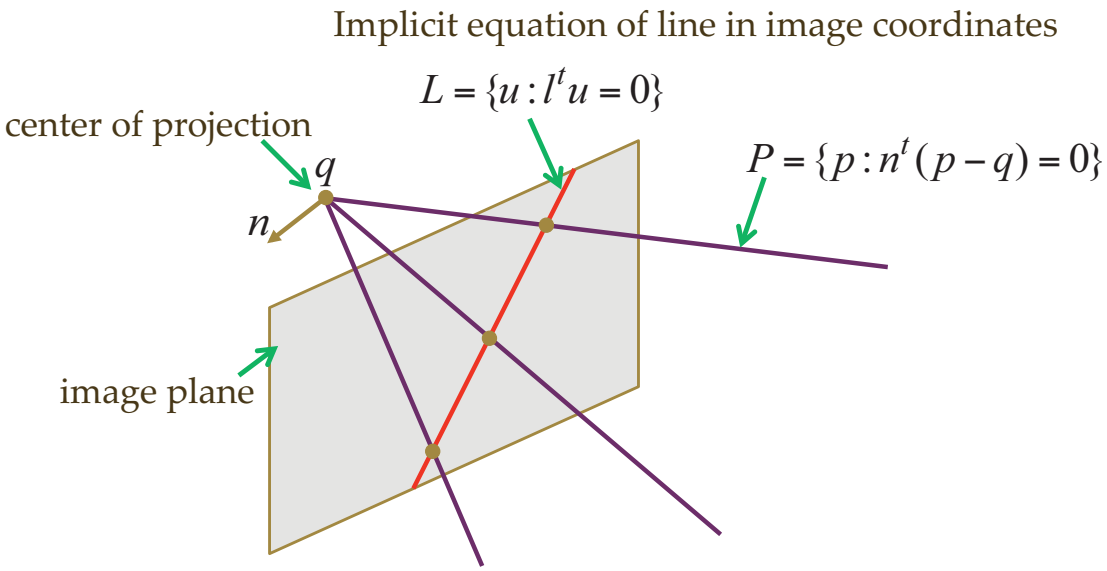
$$X_C = RX_W + T$$

intrinsic parameters

$$\lambda u = K(R p_W + T)$$

$$K = \begin{pmatrix} f s_1 & f s_\theta & o^1 \\ 0 & f s_2 & o^2 \\ 0 & 0 & 1 \end{pmatrix}$$

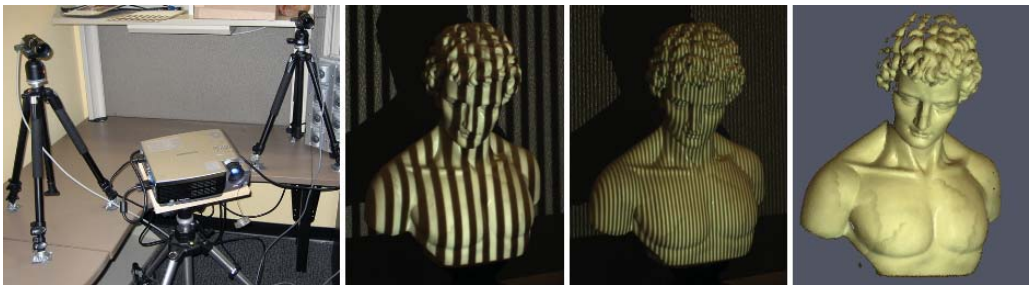
Plane Defined by Image Line and Projection Center



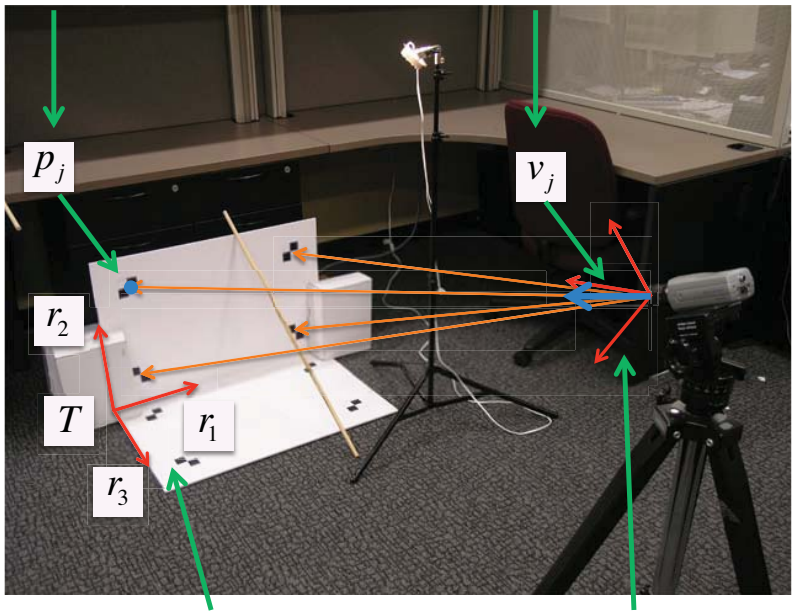
$$0 = \lambda l^t u = l^t (R p_W + T) = (R^t l)^t (p_W - (-R^t T)) .$$

Desktop 3D Photography

Structured Lighting



Estimating the equation of a plane



Given

$$\lambda_j v_j = Rp_j + T$$

$$j = 1,\ldots,N$$

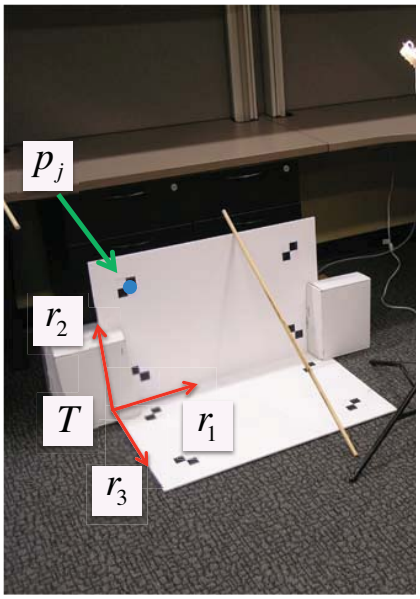
We want to estimate the rotation R and the translation T

$$R = \left[r_1 r_2 r_3\right]$$

How to solve this problem?

What is the minimum number of points necessary to solve the problem?

Estimating the equation of a plane



$$\lambda_j v_j = R\ p_j + T$$

Camera coordinate system

Plane coordinate system

Implicit equation of plane in the camera coordinate system

$$\{q : r_3^t (q - T) = 0\}$$

Parametric equation of plane in the camera coordinate system

$$\{q = T + x\ r_1 + y\ r_2 + 0\ \ r_3\} \qquad p = \left[x, y, 0\right]^t$$

Estimating the equation of two planes