Introducción a la Fotografia 3D

UBA/FCEN Marzo 27 – Abril 12 2013

Clase 8 : Jueves Abril 11

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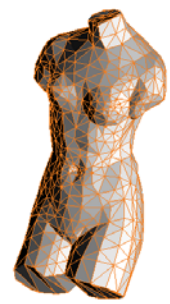
BROWN

Course Schedule

- Structured Lighting
- Projector Camera Calibration
- ***Surface Reconstruction from Point Clouds***
- Elementary Mesh Processing
- Related Projects
- Conclusion / Q & A

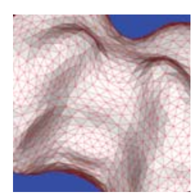
Surface Representations

- Surfaces in Mathematics
  - Parametric $S = \{p = x(u) : u = (u_1, u_2) \in R^2\}$
  - Implicit $S = \{p : f(p) = 0\}$ $f : V \to R$ $V \subset R^3$ (level set)
- We can only operate on a surface representation
  - A data structure defined by a finite number of parameters
  - Efficient to perform certain geometric operations
- Point clouds (surfaces represented as sets of samples)
  - Positions
  - Optional properties: normals, colors, etc
- Polygon meshes (piecewise planar surfaces)
  - vertices, edges, and faces
  - Optional properties: normals, color, texture coordinates, etc.

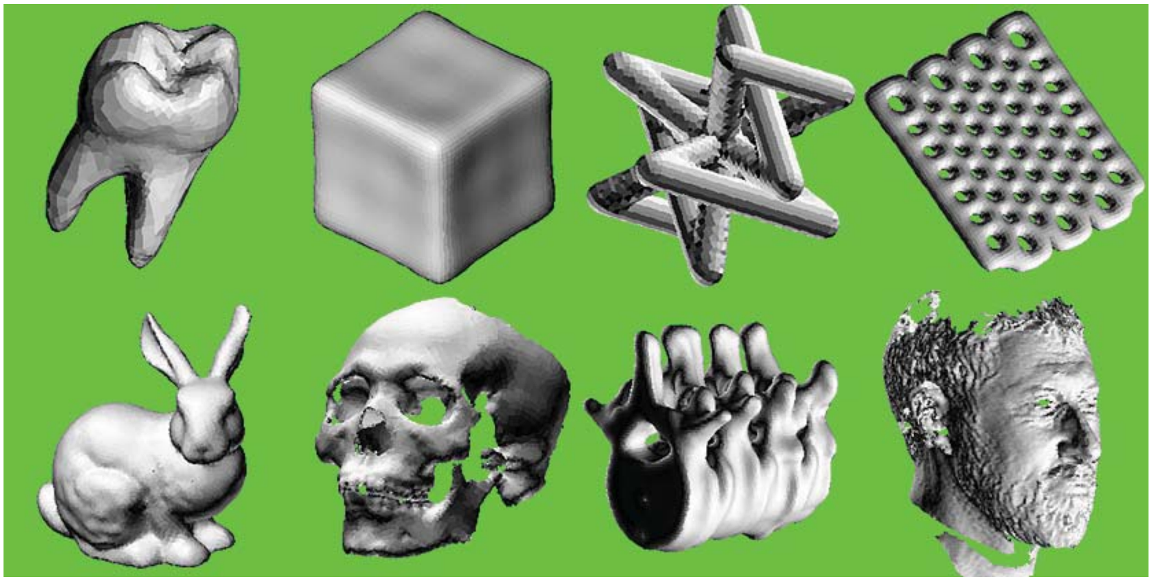


Connectivity / Watertight Surfaces

- Most applications require **connectivity information**
  - Efficient ways to find points in close proximity to each other
- Point clouds do not provide connectivity information
  - Additional data structures are needed to efficiently find neighboring points
- Connectivity is explicit in polygon meshes: **edges**
- **Triangulate** the point cloud to get connectivity information
  - Find an interpolating or approximating triangle mesh
- Many applications require **watertight** surfaces: continuous closed surfaces which partition 3D space into an **inside** and an **outside**
  - Point clouds are not watertight
  - Polygon meshes may be watertight
- Will the triangulation constructed from the point cloud be watertight ?

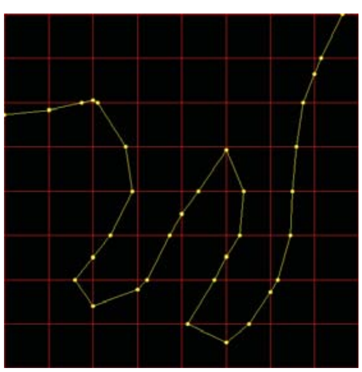


Polygon Meshes



Surface Reconstruction from Point Clouds

- Every regular **Implicit surface** is watertight $S = \{p : f(p) = 0\}$
- An **Isosurface** is a polygonal approximation of an implicit function associated with a volumetric grid



Surface Reconstruction from Point Clouds

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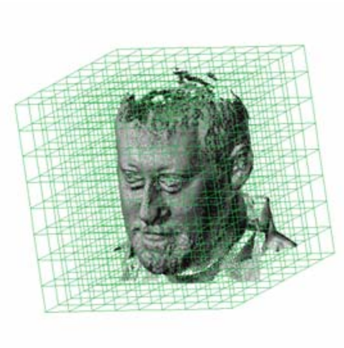
- **Marching Cubes** is an algorithm to compute an isosurface from an implicit surface evaluated on the vertices of a regular hexahedral grid



W.E. Lorensen, H.E. Cline. Marching Cubes:A high resolution 3D surface reconstruction algorithm. Siggraph,1987

Surface Reconstruction from Point Clouds

- Every regular **Implicit surface** is watertight $S = \{p : f(p) = 0\}$
- An **Isosurface** is a polygonal approximation of an implicit function associated with a volumetric grid
- **Marching Cubes** is an algorithm to compute an isosurface from an implicit surface evaluated on the vertices of a regular hexahedral grid
- Similar simple algorithms exists to generate isosurfaces from an implicit function evaluated on the vertices of a tetrahedral grid
- We will only discuss here approximation algorithms to fit implicit surfaces to point clouds
- Algorithms related to the Poisson Equation

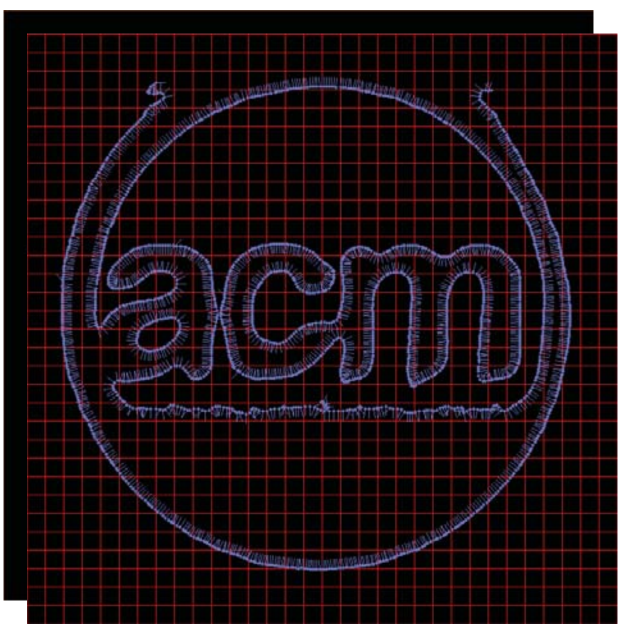
W.E. Lorensen, H.E. Cline. Marching Cubes:A high resolution 3D surface reconstruction algorithm. Siggraph,1987

M. Kazhdan, M. Bolitho, H. Hoppe. Poisson Surface Reconstruction. European Symposium on Geometry Processing, 2006

Curve Reconstruction from Point Clouds

- Oriented points

Curve Reconstruction from Point Clouds

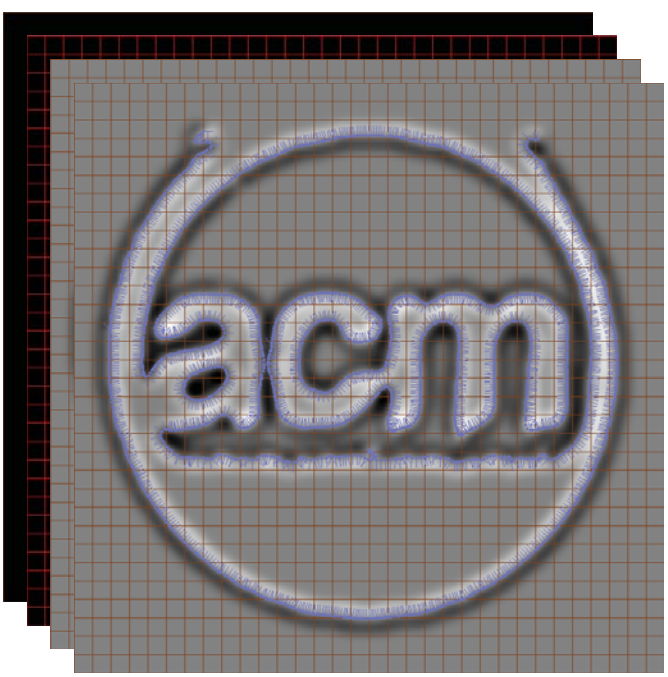


- Oriented points
- Regular grid

Curve Reconstruction from Point Clouds

- Oriented points
- Regular grid
- Implicit function

Curve Reconstruction from Point Clouds



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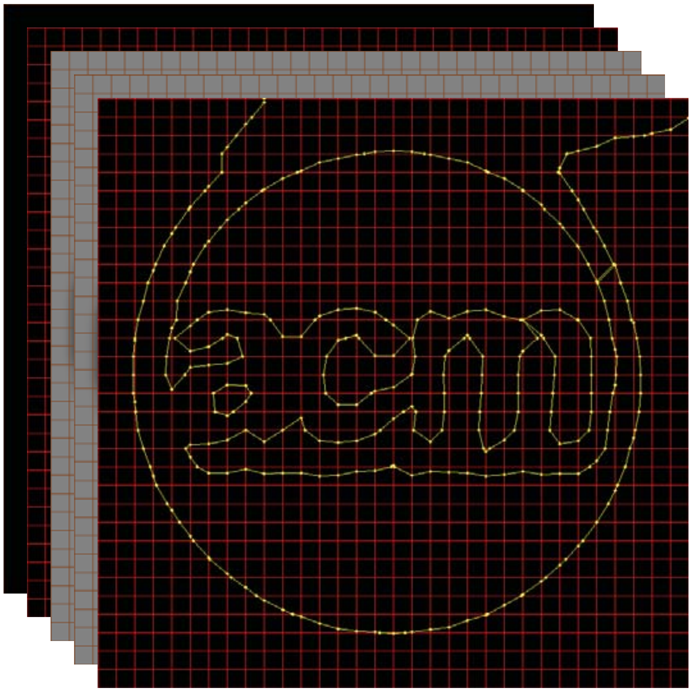
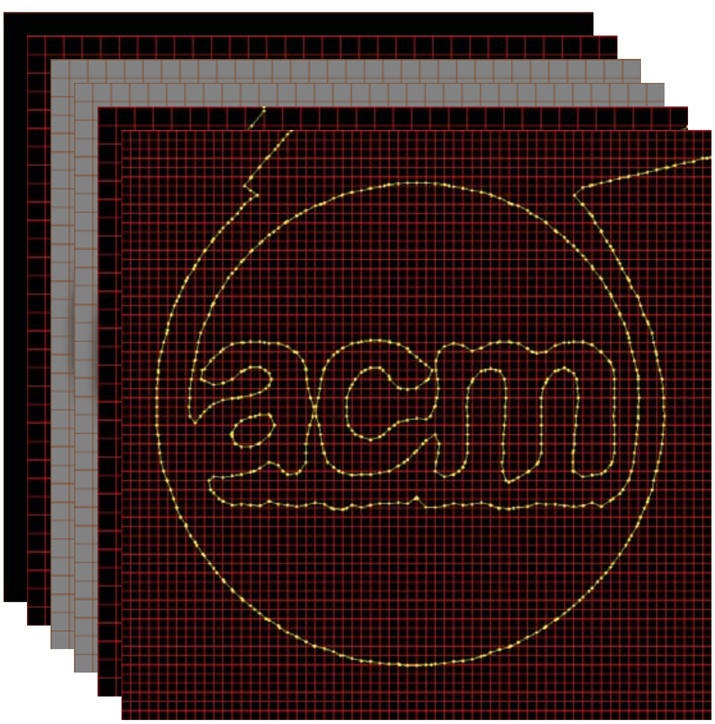
Curve Reconstruction from Point Clouds



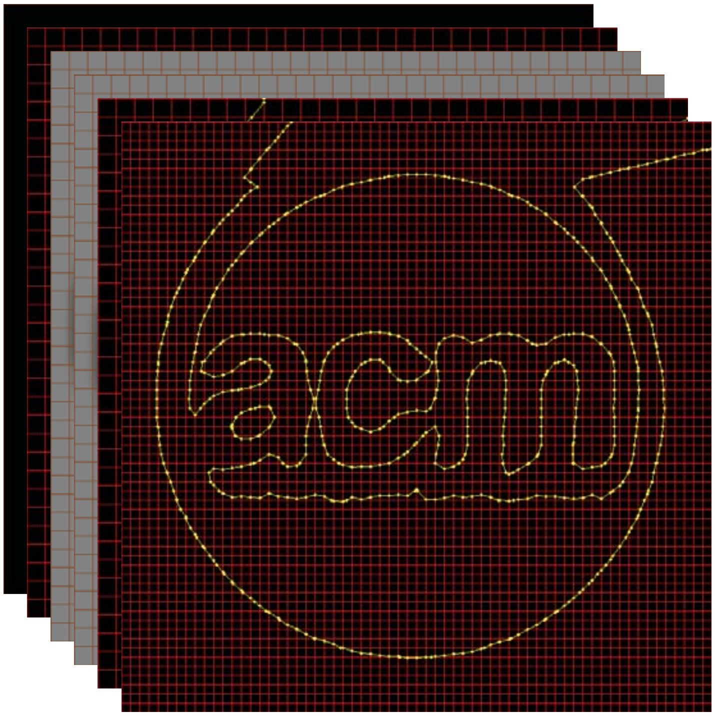
- Oriented points
- Regular grid
- Implicit function
- Isocurve

Curve Reconstruction from Point Clouds



- Oriented points
- Regular grid
- Implicit function
- Isocurve
- Grid too coarse: Aliasing
- Finer grid resolves topology

Curve Reconstruction from Point Clouds



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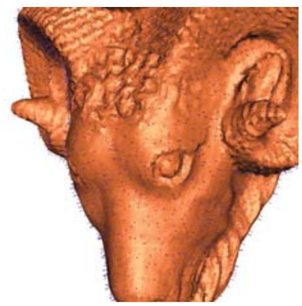
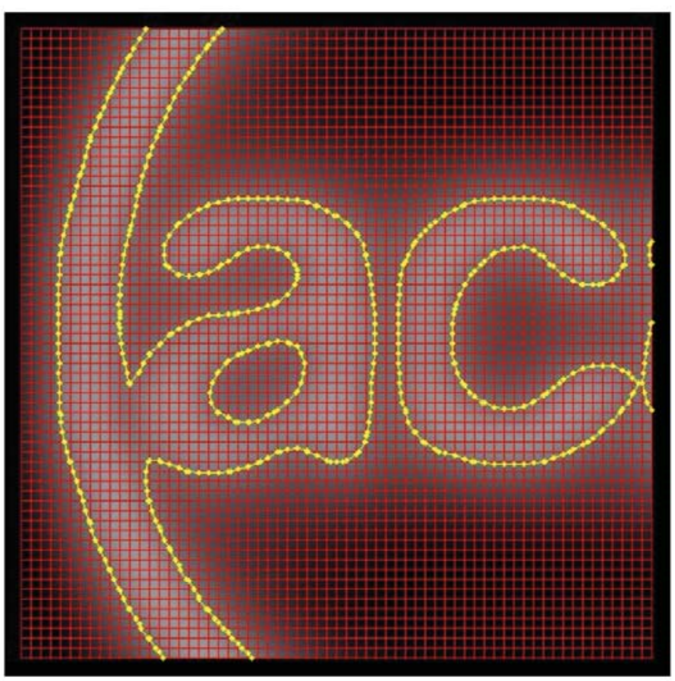


IsoCurves



- Given a continuous function

  $$f(x_1, x_2)$$

- Sampled on a regular grid

  $$G = (V, E, C)$$

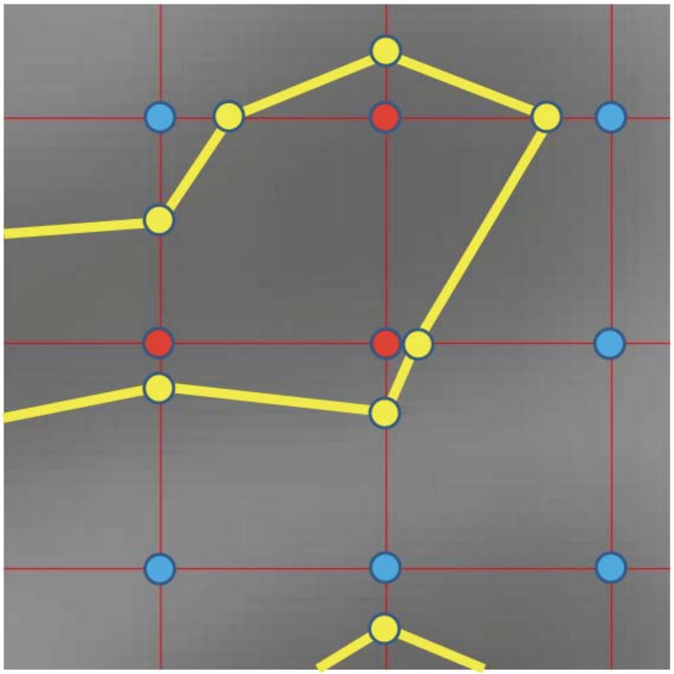
  $$F = \{f_v : v \in V\}$$

- Compute a polygonal approximation of a level set

  $$\mathcal{C}_\lambda = \{x : f(x) = \lambda\}$$

- Increase grid resolution if necessary

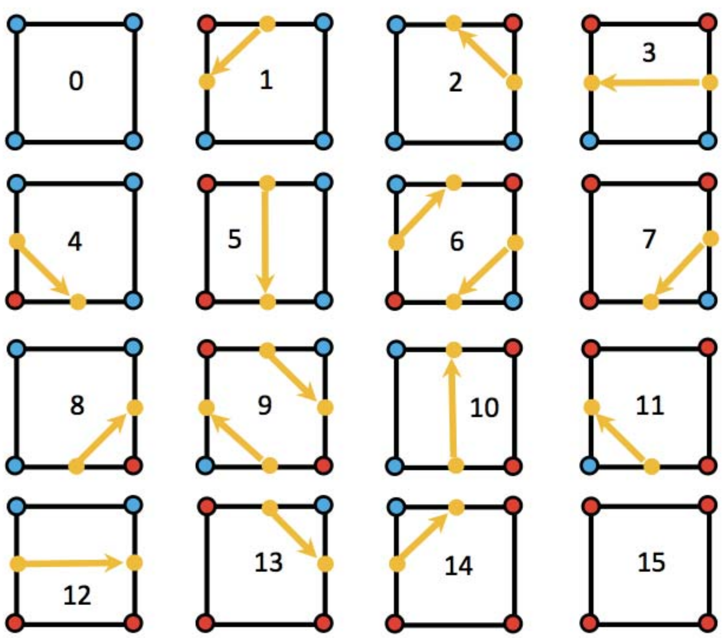
The Marching Lines Algorithm (ML)



4 STEPS

1. Determine grid vertex sign bits
2. Determine supporting grid edges
3. Compute location of Isovertices along supporting grid edges
4. Interconnect isovertices by table look-up within each cell

The Marching Lines Table



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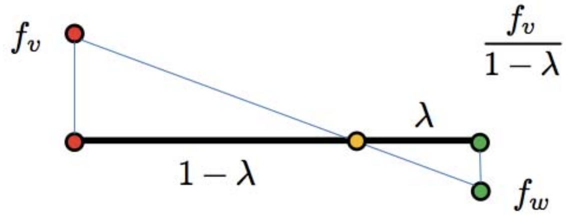
Choices for 6 & 9

The 4 Steps

1. Determine grid vertex sign bits $b_v = \begin{cases} 1 & f_v \geq 0 \\ 0 & f_v < 0 \end{cases}$

2. Determine supporting grid edges  $b_v \neq b_w$

3. Compute location of IsoVertices along supporting grid edges



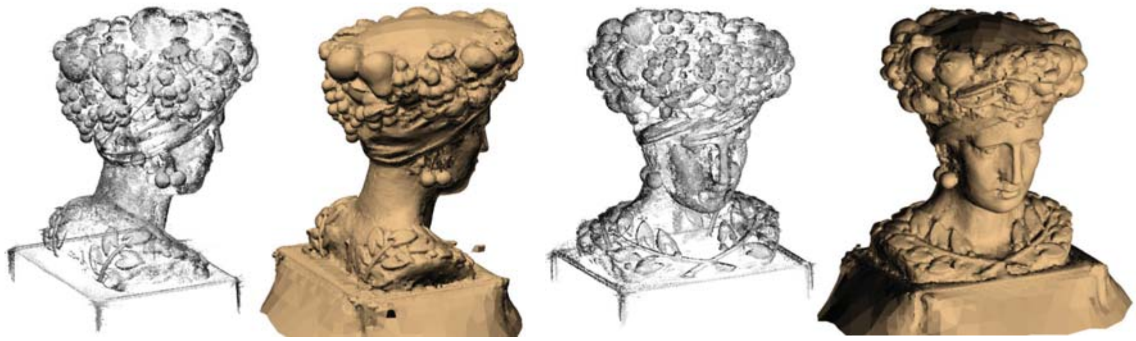
$$\frac{f_v}{1-\lambda} + \frac{f_w}{\lambda} = 0 \quad \Leftrightarrow \quad \lambda = \frac{f_w}{f_w - f_v}$$

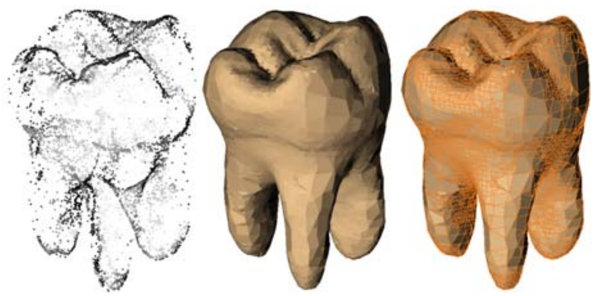
4. Interconnect IsoVertices by table look-up within each cell

Related Papers & Projects

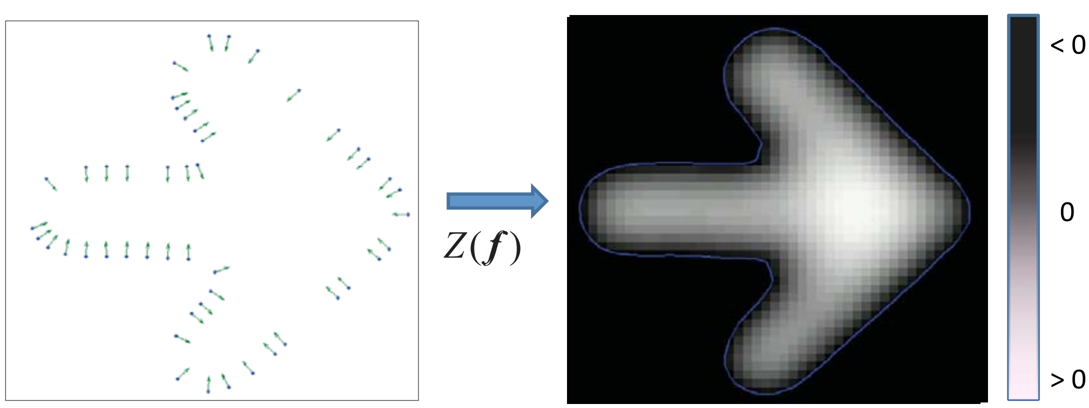
- Vector Field Isosurface-Based Reconstruction From Oriented Points, by Sibley & Taubin, Siggraph 2005 (Sketch).
- **Smooth Signed Distance Surface Reconstruction, by Calakli & Taubin, PG 2011 & Computer Graphics Forum 2011.**
- Smooth Signed Distance Colored Surface Reconstruction, by Calakli & Taubin, chapter in State-of-the-Art Volume on Computer Graphics, Visualization, Visual Analytics, VR and HCI, 2012.
- Accurate 3D Footwear Impression Recovery from Photographs, by Andalo, Calakli, Taubin, and Goldenstein, Proceedings of the 4th. International Conference on Imaging for Crime Detection and Prevention (ICDP-2011).
- High Resolution Surface Reconstruction from Multi-view Aerial Imagery, Calakli, Ulusoy, Restrepo, Mundy & Taubin, 3DIMPVT 2012
- REVEAL Digital Archaeology Project
- Cuneiform Automatic Translation Project

Particularly Good at Extrapolating Missing Data





Implicit function framework



Oriented Points, ***D***
(samples from unknown surface ***S***)

Computed Implicit Surface, ***S'***

Find a scalar valued function $f : D \rightarrow \Re$, whose zero level set ***Z(f) = S'*** is the estimate for true surface ***S***

Implicit Curve and Surface Reconstruction

- Input: oriented point set:

  $D = \{ (\mathbf{p}_i, \mathbf{n}_i) \; i=1,\ldots,N\}$

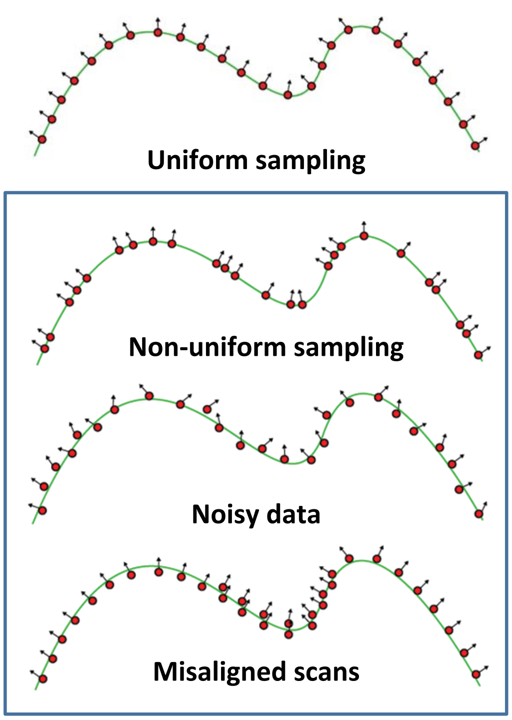
  contained in a bounding volume V
- Output: implicit surface

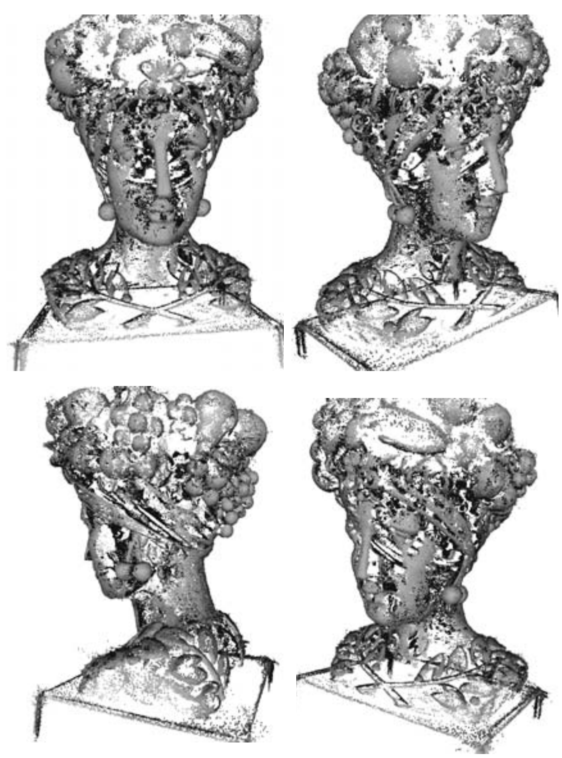
  $S = \{ \mathbf{x} \mid f(\mathbf{x}) = 0 \}$

  with the function defined on V, such that

  $f(\mathbf{p}_i) = 0 \quad \text{and} \quad \nabla f(\mathbf{p}_i) = \mathbf{n}_i \quad \forall (\mathbf{p}_i,\mathbf{n}_i) \in D$
- A family of implicit functions with a finite number of parameters has to be chosen
- Parameters must be estimated so that the conditions stated above are satisfied, if not exactly, then in the least-squares sense

Challenges





General Approaches

- Interpolating polygon meshes
  Boissonnat [1984], Edelsbrunner [1984]
  Amenta et al. [1998], Bernardini et al. [1999]
  Dey et al. [2003][2007], …
- Implicit function fitting
  Taubin [1991], Hoppe et al. [1992], Curless et al. [1996]
  Whitaker [1998], Carr et al.[2001], Davis et al. [2002],
  Ohtake et al. [2004], Turk et al. [2004], Shen et al. [2004]
  Sibley-Taubin [2005]

Poisson Surface Reconstruction

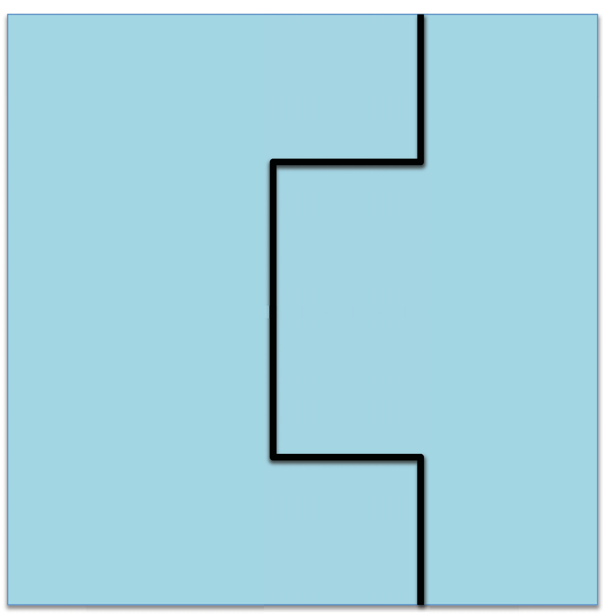
Kazhdan et al. [2006]

Manson et al. [2008]

Poisson Surface Reconstruction

1. Extend oriented points to continuous vector field $v(p)$ defined on the whole volume, so that
$$v(\mathbf{p_i}) \approx n_\mathbf{i}$$
2. Integrate vector field, by minimizing
$$\int_V \|\nabla f(p) - v(p)\|^2 \, dp$$
3. Determine isolevel, by minimizing
$$\sum_{i=1}^{N} (f(\mathbf{p_i}) - f_0)^2$$

Main problem with this approach



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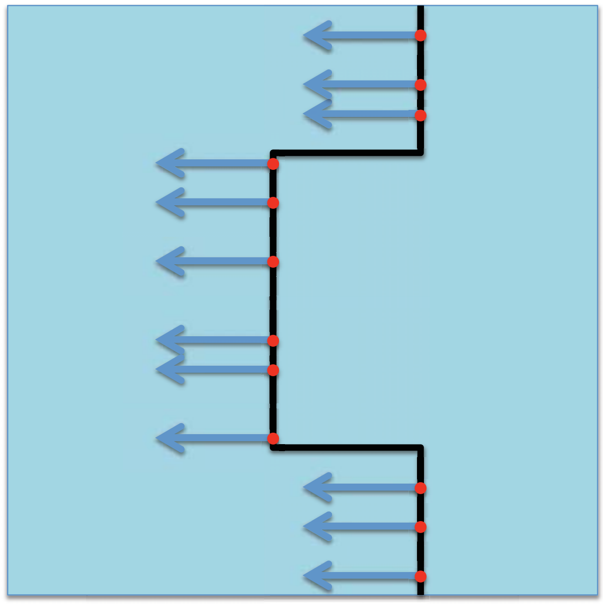

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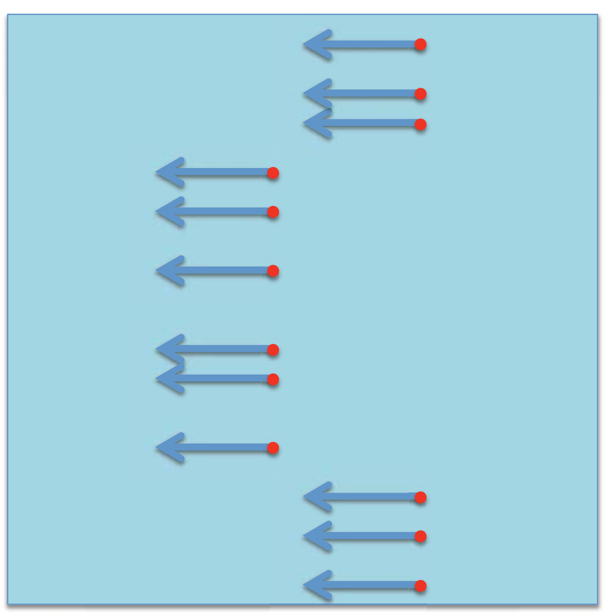

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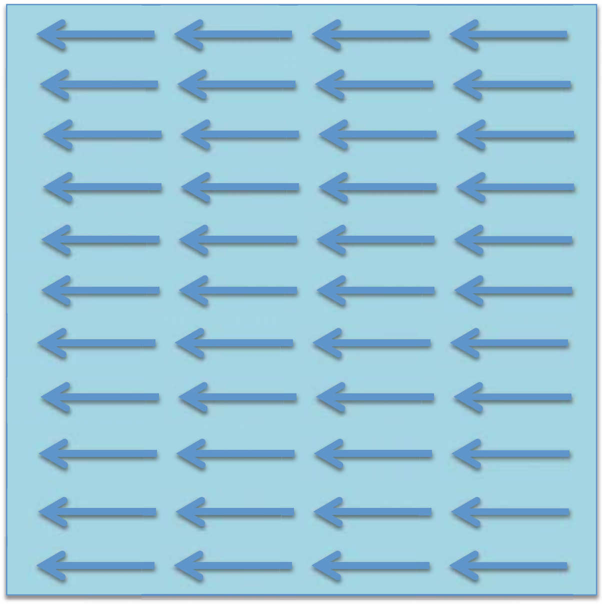

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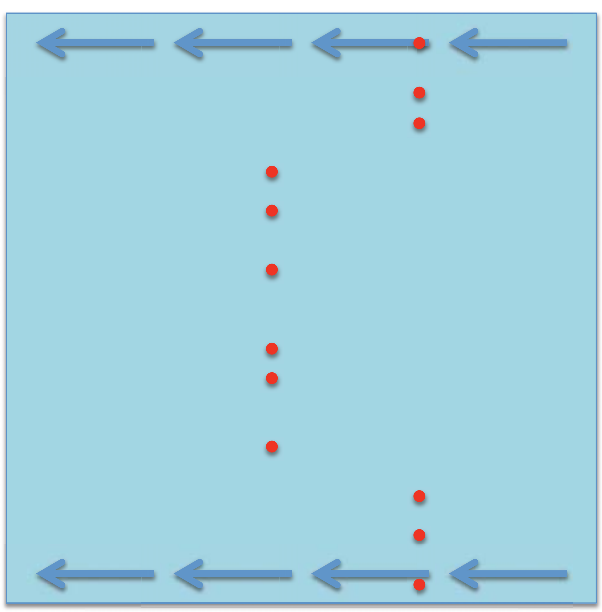

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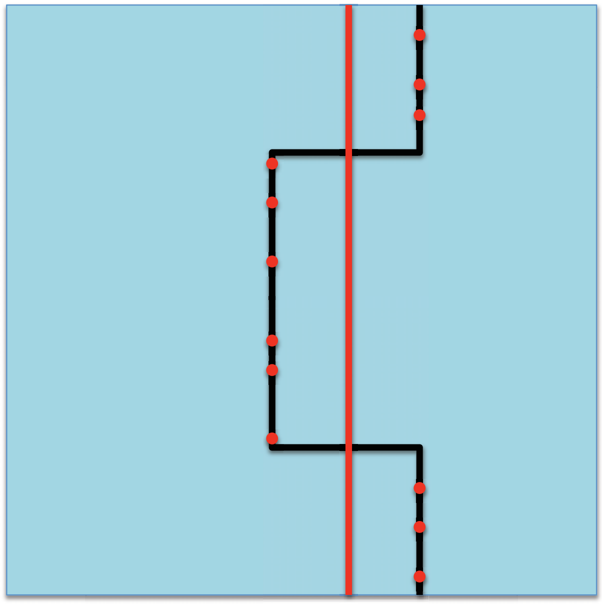
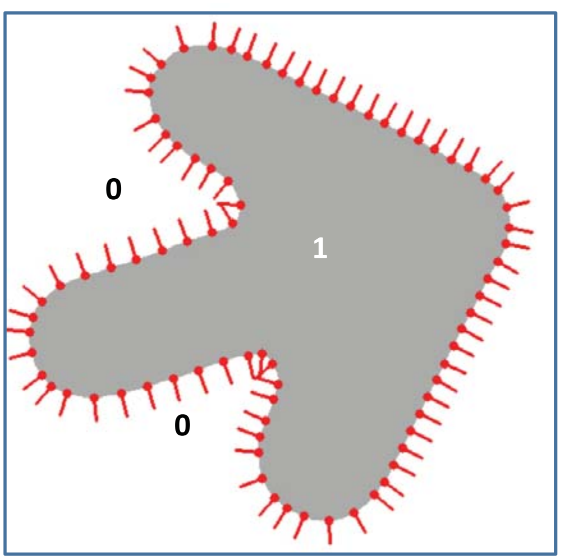
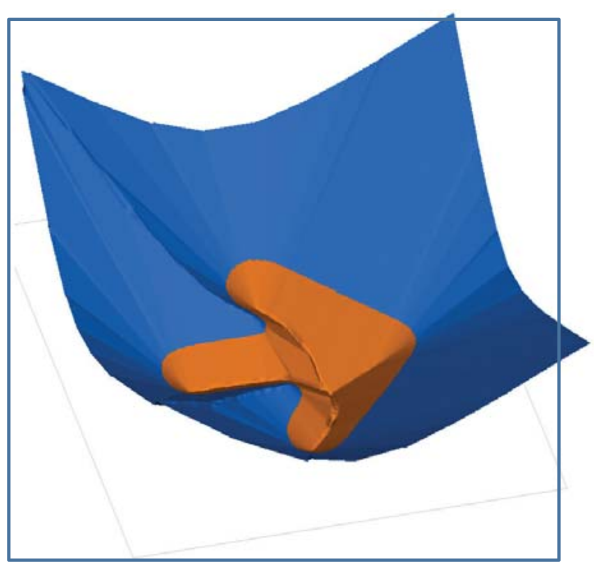


What kind of implicit function?



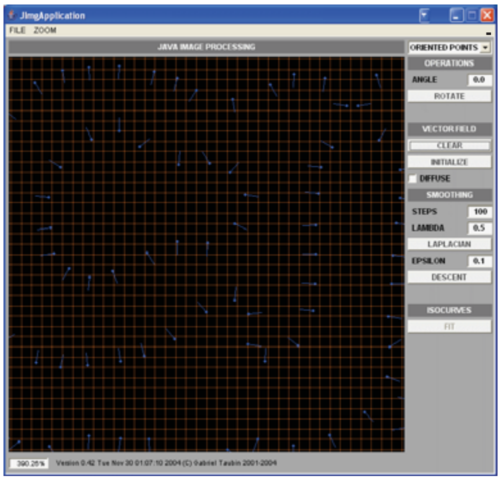
Indicator Function

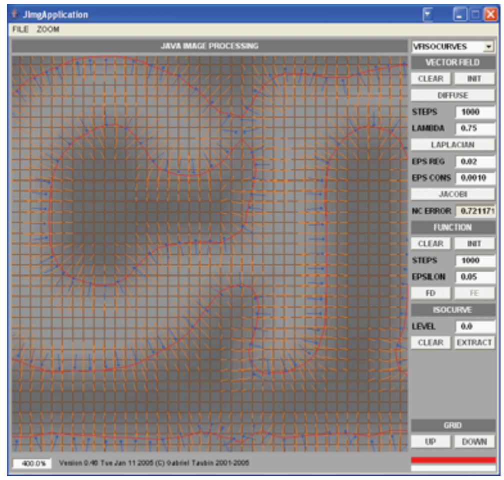


Smooth Signed Distance Function

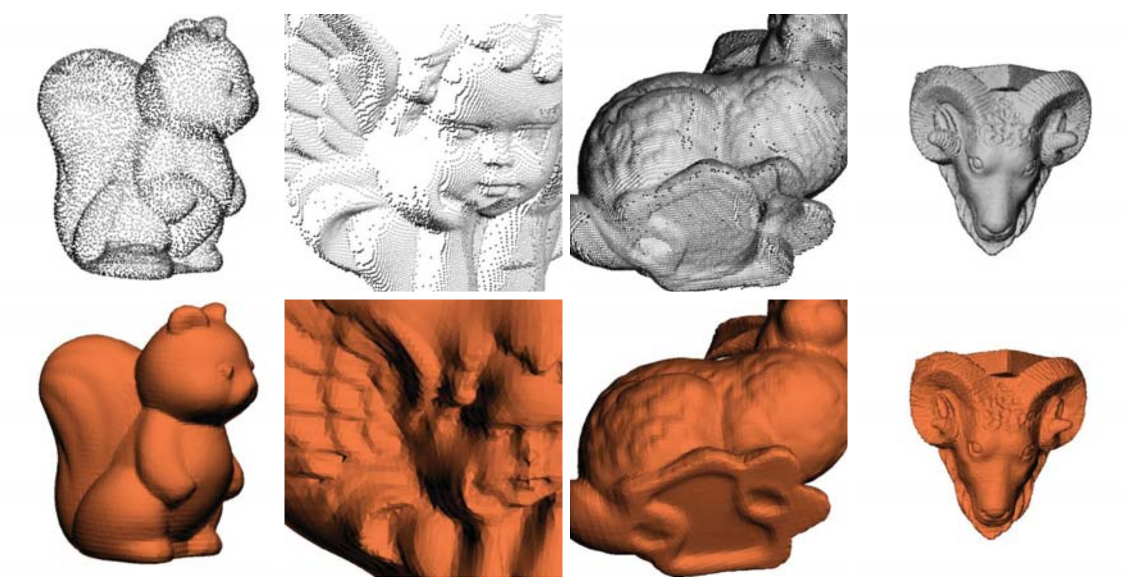
Vectorfield Isosurface-Based Reconstruction From Oriented Points
P. Sibley and G. Taubin [Siggraph 2005 Sketch]

- Surface reconstruction from cloud of oriented points
- Implicit representation can deal with missing data
- Rather than fitting analytic function (RBFs, etc), and then extract isosurface for visualization, fit isosurface directly to data

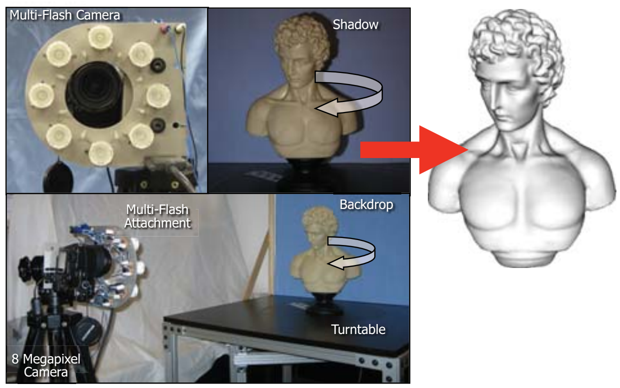


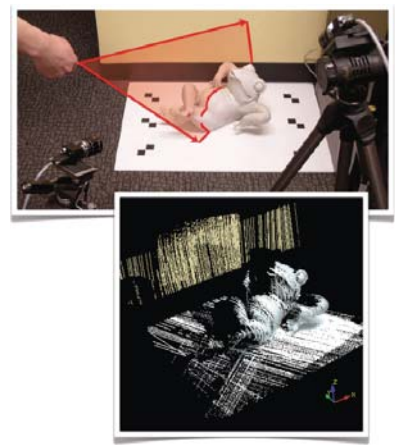


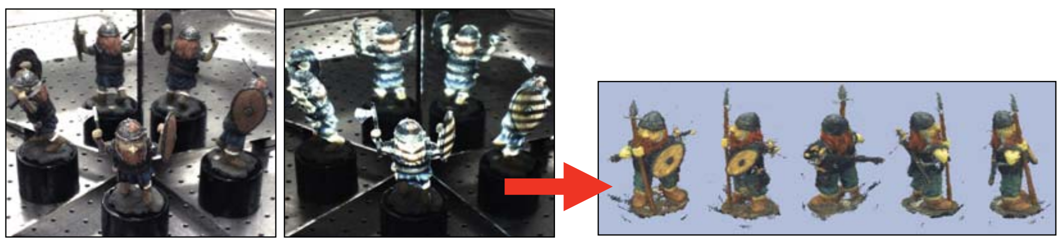
Squirrel(9K) Angel(24K) Bunny(35K) Ram(678K)



Some Methods to Capture 3D Point Clouds







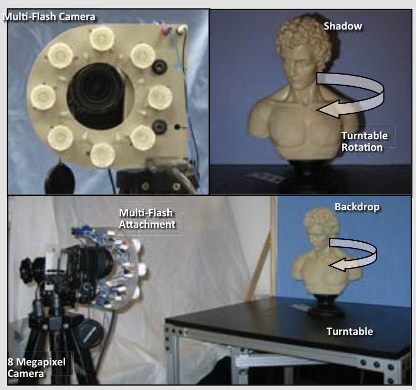
Beyond Silhouettes: Surface Reconstruction using Multi-Flash Photography

D. Crispell, D. Lanman, P. Sibley, Y. Zhao and G. Taubin [3DPVT 2006]

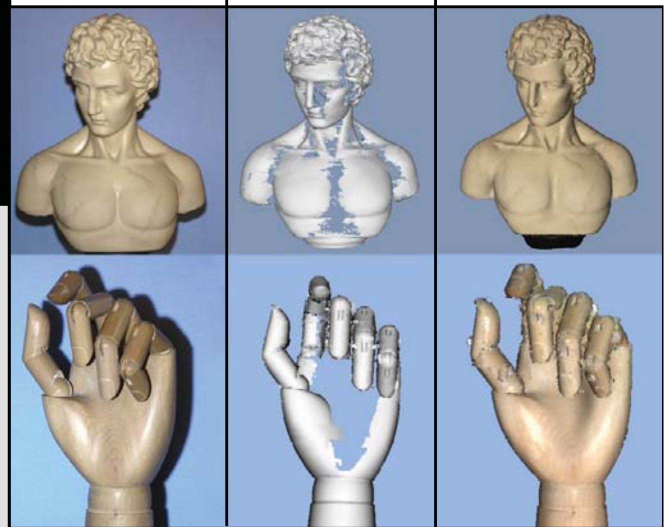
Multi-Flash 3D Photography: Capturing the Shape and Appearance of 3D Objects

A new approach for reconstructing 3D objects using shadows cast by depth discontinuities, as detected by a multi-flash camera. Unlike existing stereo vision algorithms, this method works *even with plain surfaces*, including unpainted ceramics and architecture.

Data Capture: A turntable and a digital camera are used to acquire data from 670 viewpoints. For each viewpoint, we capture a set of images using illumination from four different flashes. Future embodiments will include a small, inexpensive **handheld multi-flash camera**.



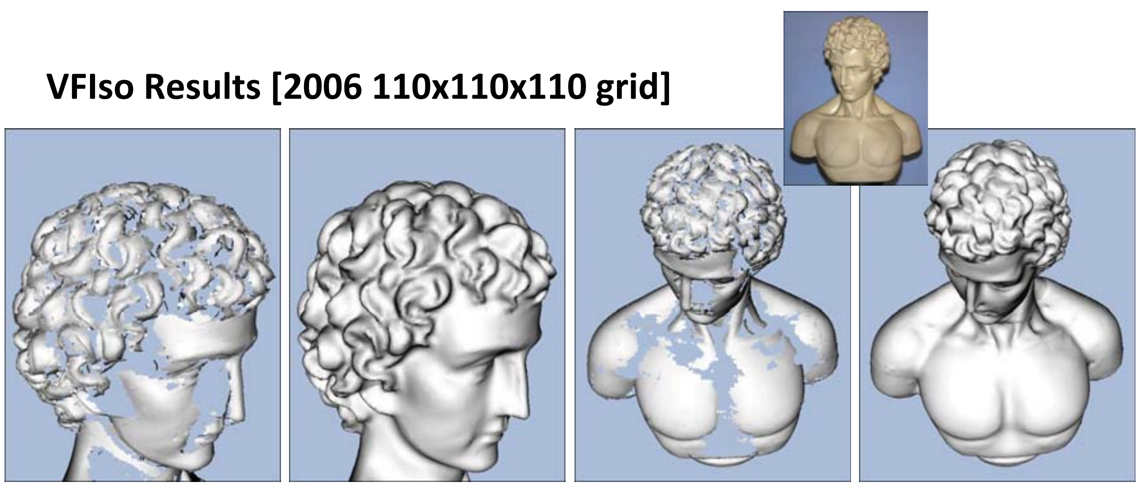
Multi-Flash Turntable Sequence: Input Image	Estimated Shape: 3D Point Cloud	Recovered Appearance: Phong BRDF Model

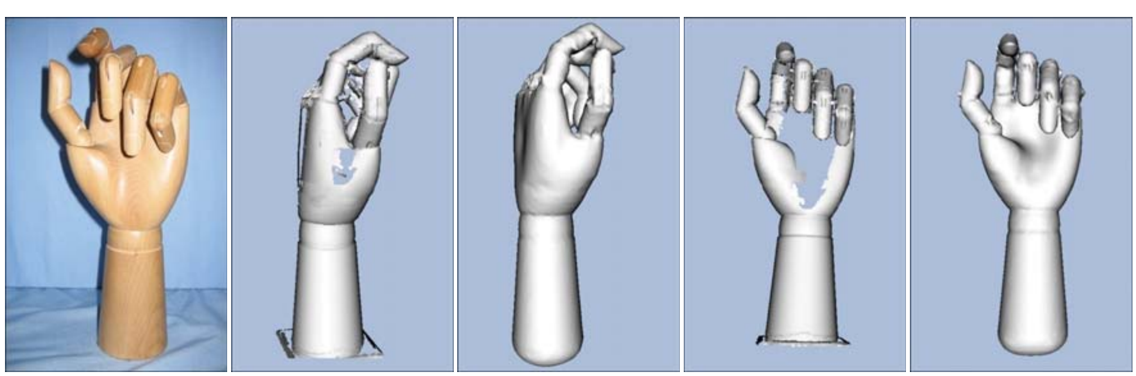


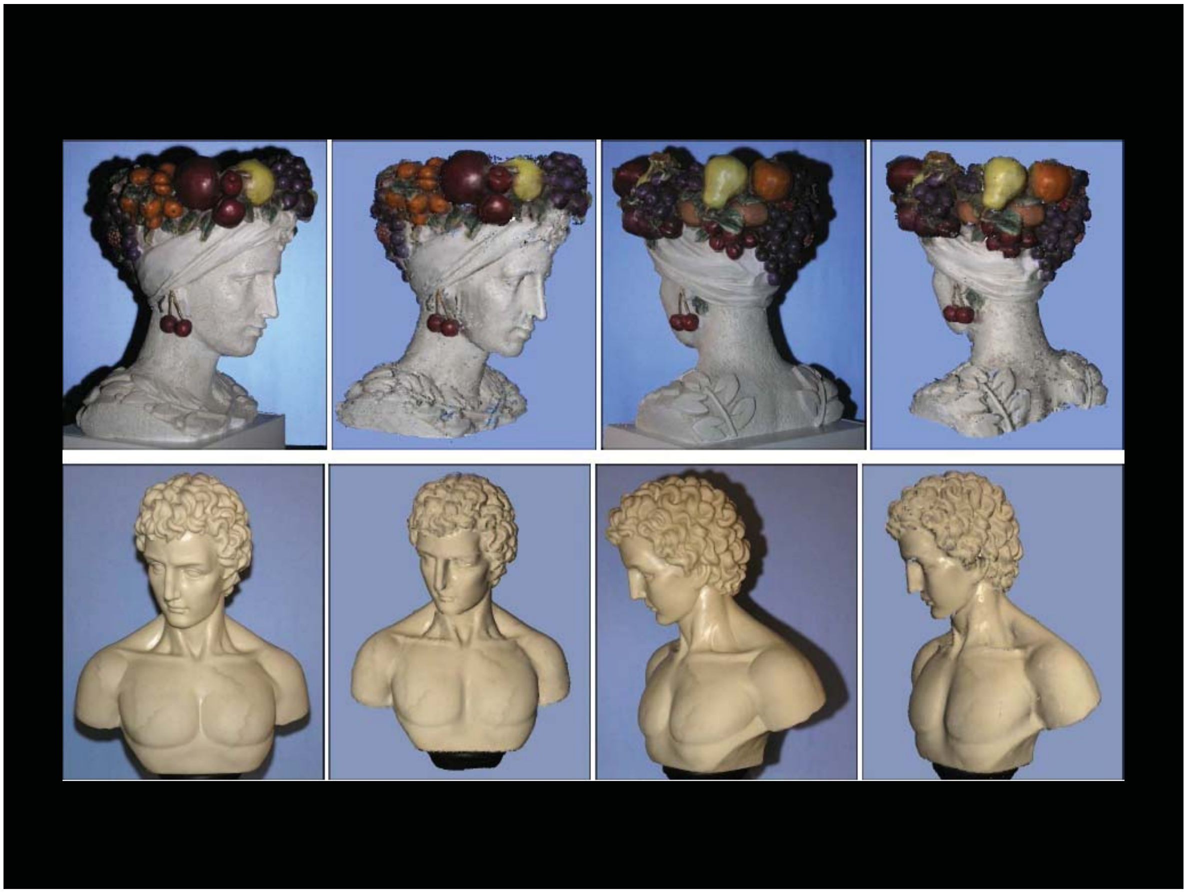
Recovering a Smooth Surface

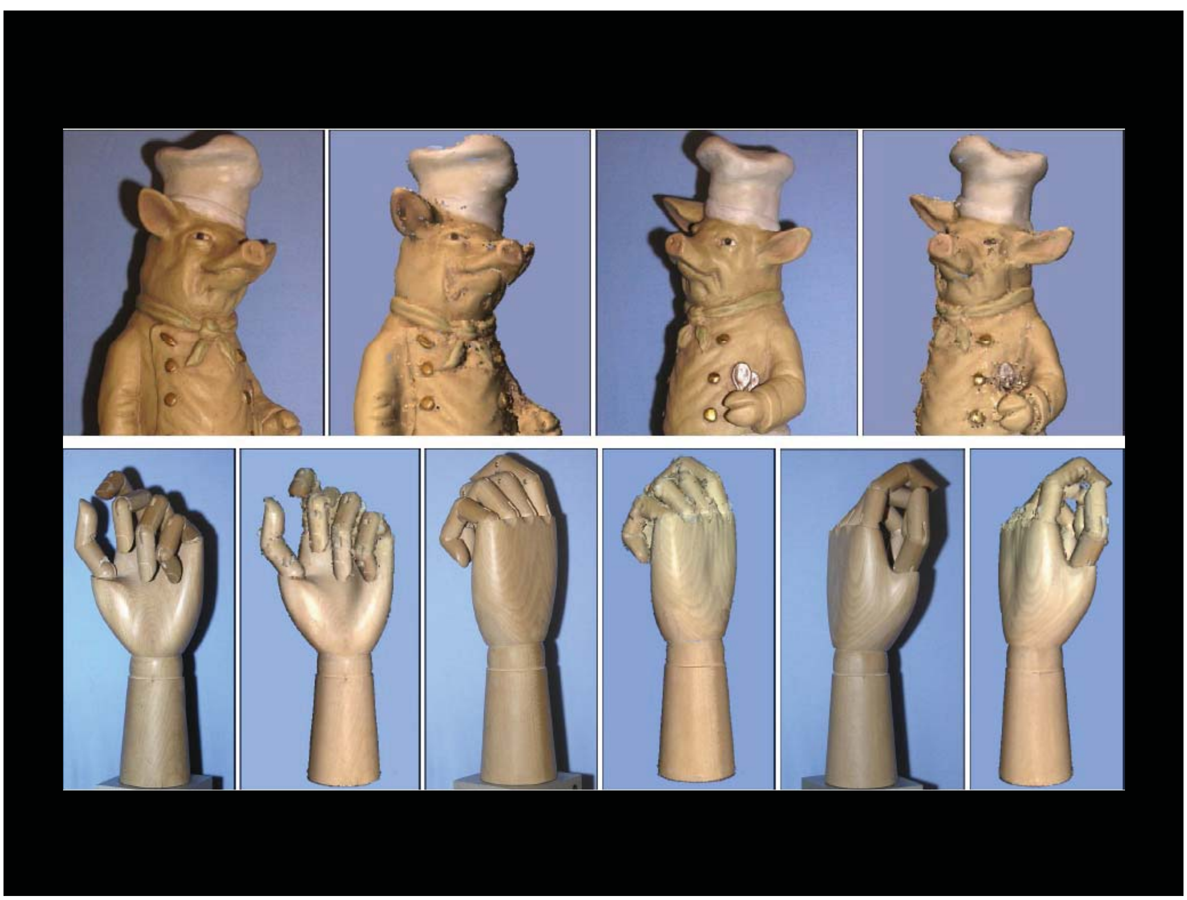
The reconstructed point cloud can possess errors, including gaps and noise. To minimize these effects, we find an implicit surface which interpolates the 3D points. This method can be applied to any 3D point cloud, including those generated by laser scanners.

VFIso Results [2006 110x110x110 grid]









SSD Continuous Formulation

- Oriented point set:

  $D = \{ (\mathbf{p_i}, \mathbf{n_i}) \}$ sampled from a surface S

- Implicit surface:

  $S = \{ \mathbf{x} \mid f(\mathbf{x}) = 0 \}$ such that

  $f(\mathbf{p_i}) = 0$ and $\nabla f(\mathbf{p_i}) = \mathbf{n_i} \;\; \forall (\mathbf{p_i},\mathbf{n_i}) \in D$

- Least squares energy:

$$E(f) = \sum_{i=1}^{N} f(\mathbf{p_i})^2 + \lambda_1 \sum_{i=1}^{N} \left\| \nabla f(\mathbf{p_i}) - \mathbf{n_i} \right\|^2 + \lambda_2 \int_V \| \mathrm{H} f(\mathbf{x}) \|^2 \, d\mathbf{x}$$

What does the regularization term do ?

$$\frac{\lambda_0}{N} \sum_{i=1}^{N} f(p_i)^2 + \frac{\lambda_1}{N} \sum_{i=1}^{N} \|\nabla f(p_i) - n_i\|^2 + \frac{\lambda_2}{|V|} \int_V \|Hf(x)\|^2 \, dx$$

$$Hf(x) = \left[\frac{\partial \nabla f(x)}{\partial x_1} \;\; \frac{\partial \nabla f(x)}{\partial x_2} \;\; \frac{\partial \nabla f(x)}{\partial x_3} \right]$$

- Near data points: since the data terms dominate, the function approximates the signed distance
- Away from data points: the regularization term dominates and forces the gradient to be smooth and close to constant

Role of each energy term

$$E(f,v,M)=\sum_{i=1}^{N} f(\mathbf{p_i})^2+\lambda_1\sum_{i=1}^{N}\left\|v(\mathbf{p_i})-\mathbf{n_i}\right\|^2+\lambda_2\int_V \| M(\mathbf{x})\|^2\,d\mathbf{x}$$

Quadratic energy in f, v, and M

If f, v, and M are **linear functions of the same parameters**, then the minimization reduces to a least squares problem

Linear families of functions

$$f(x)=\sum_{\alpha\in\Lambda} f_\alpha\,\phi_\alpha(x)=\Phi(x)^t F$$

- Popular Smooth Basis Functions
  - Monomials [Taubin'91]
  - Radial basis functions [Carr et al., '01],
  - Compactly supported basis functions [Othake et al. '04],
  - Trigonometric polynomials [Kazhdan et al. '05],
  - B-splines [Kazhdan et al., 06],
  - Wavelets [Manson et al. '08],

Non-homogenous, Quadratic energy

$$E(F)=F^t AF-2b^t F+c$$

Global minimum

$$AF=b$$

We can use Independent Discretizations

- Hybrid FE/FD discretization
  - Trilinear interpolant for the function $f(\mathbf{x})$
  - Primal finite differences for the gradient $\nabla f(\mathbf{x})$
  - Dual finite differences for the Hessian $Hf(\mathbf{x})$

- As long as $f(\mathbf{x})$, $\nabla f(\mathbf{x})$, and $Hf(\mathbf{x})$ are written as a linear combinations of **the same** parameter vector F

Non-homogenous, Quadratic energy

$$E(F) = F^t AF - 2b^t F + c$$

Global minimum

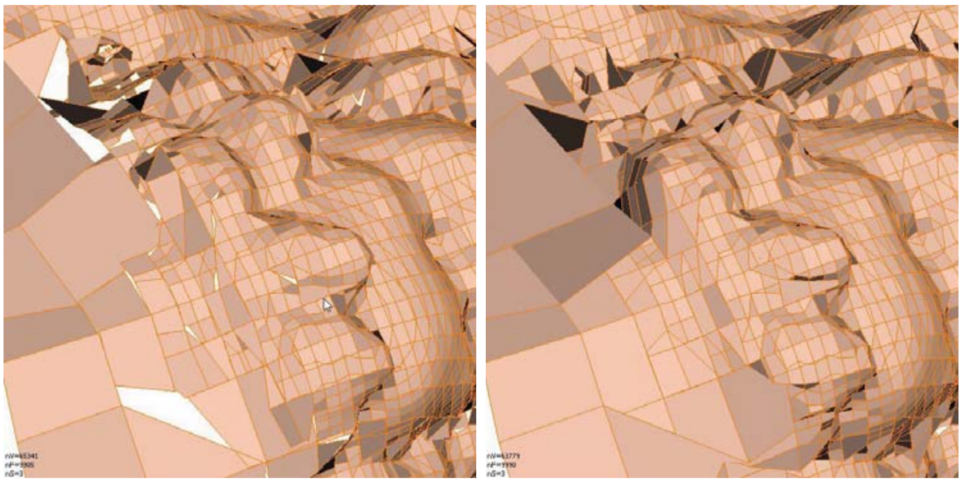
$$AF = b$$

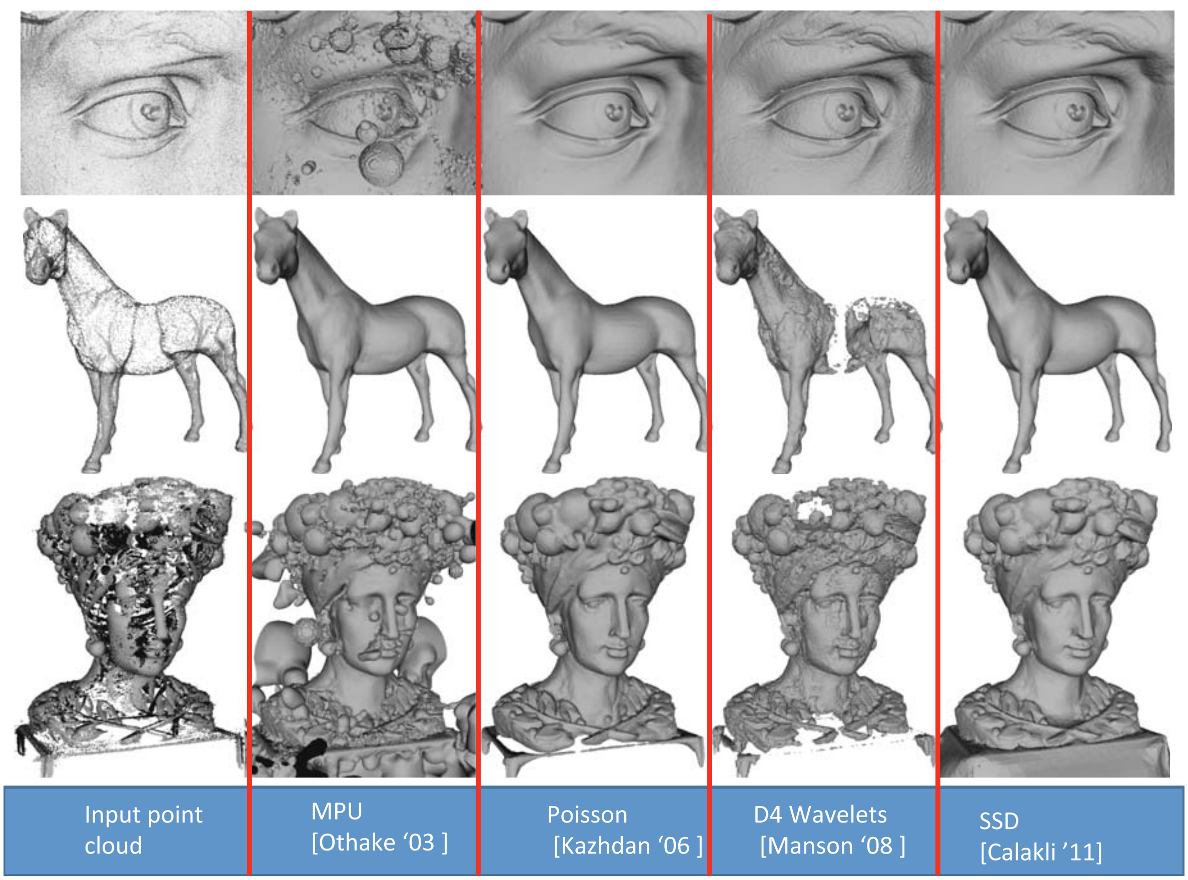
Implementation

- Primal-Dual octree data structure
- Cascading multi-grid iterative solver (conjugate gradient):
  Solve the problem on a much coarser level
  - Use the solution at that level to initialize the solution at the next level
  - Refine with the iterative solver
- Iso-surface extraction (crack-free)
  - Dual marching cubes [Schaefer 2005]

Marching Cubes on Octrees

- Non-conforming hexahedral mesh
- Results in crack problem.
- Problem solved by Dual Marching Cubes





SSD Surface Reconstruction

- Theoretical contributions:
  - Oriented point samples regarded as samples of Euclidean signed distance function
  - Reconstruction as global minimization problem
  - Yet sparse system of linear equations
- Empirical advantages:
  - Robust to noise and uneven sampling density
- Future work:
  - Streaming out-of-core implementations
  - Parallel/Multi-core/GPU implementations
  - Dynamic shapes

Course Schedule

- Structured Lighting
- Projector-Camera Calibration
- Surface Reconstruction from Point Clouds
- ➢ **Elementary Mesh Processing**
- Related Projects
- Conclusion / Q & A