DOUBLY-LINKED HALF-EDGE DATA STRUCTURE

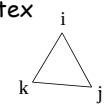
3D Photography and Geometry Processing
Brown Spring 2008
Gabriel Taubin

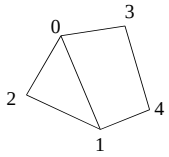
3D Representations

- Surfaces
  - Polygonal meshes
    - No connectivity (3 3D vertices per triangle)
    - IndexedFaceSet (VRML file format)
    - Half-Edge data structure (manifold meshes)
  - Boundaries of solid objects
- Volumes (solid objects)
  - Implicit surfaces
    - inside-outside boundary
  - How to convert to polygonal mesh

IndexedFaceSet

- Array of vertex coordinates
- Each 3D vertex has an associated vertex index in {0,…,V-1}
- A triangle is defined by three vertex indices (i,j,k)
- A polygonal face without holes is defined by more indices
- coordIndex [0,1,2,-1,0,3,4,1,-1]
- VRML'97 file format





Polygonal Mesh Components

- Connectivity
  - coordIndex (faces)
- Geometry
  - coord (vertex coordinates)
- Properties
  - color/colorIndex/colorPerVertex
  - normal/normalIndex/normalPerVertex
  - texCoord/texCoordIndex

Connectivity

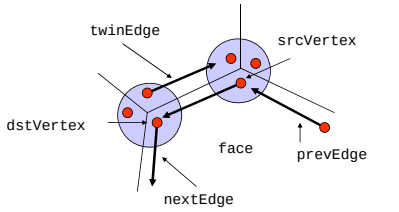
- Edges
  - Boundary (1 incident face)
  - Regular (2 incident faces)
  - Singular (3 or more incident faces)
- Vertices
  - Regular / Singular
- Connected components
  - Connected Components of Dual Graph

Manifold Meshes

- No singular edges
  - Boundary
    - 1 incident face
  - Regular
    - 2 incident faces
- No singular vertices
  - Boundary
    - dual graph of set of incident faces form a path
  - Regular
    - dual graph of set of incident faces form a cycle
- Data Structure to represent and operate ?

Doubly-linked data structure

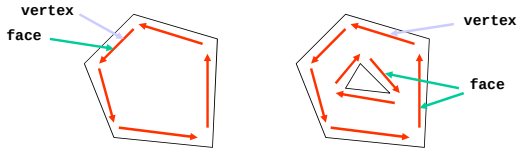
- Planar subdivisions
- Orientation
- Vertices / Faces / Half-Edges




```
halfEdge {
  face
  srcVertex
  nextEdge
  prevEdge
  twinEdge
}
```

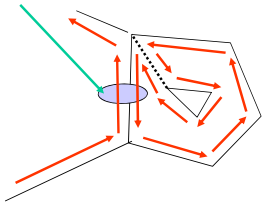
Doubly-linked data structure

- One half-edge per corner of mesh
- Simple Face
  - closed loop of half-edges
- Multiply connected face
  - 1 external loop + one or more internal loops



Orientation

- Consistent if edge is added or removed
- Counterclockwise for outer loop
- Clockwise for inner loops
- Twin edges have opposite orientations



Doubly-linked data structure

- Operations
  - Traversal / triangle strips / compression
  - Surgery / Euler operations
  - Simplification / subdivision
- How to construct from IndexedFaceSet ?
  - Simply connected faces
  - Geometric intersections ignored
- Conversion to manifold
  - Removal of singular edges and vertices

How to construct from IndexedFaceSet ?

- 0) First use original vertex indices
  - 0,…,V-1
- 1) Construct all the half-edges
  - Traverse coordIndex array
  - For each face
    - One half-edge per corner
    - Set face, srcVertex, nextEdge and prevEdge
    - Set twinEdge to NULL

How to construct from IndexedFaceSet ?

- 2) Determine regular edges by counting incident faces per edge
  - Use symmetric sparse matrix data structure m with operations m.get(i,j) and m.set(i,j,value)
    - Initialize to 0
    - For each half-edge connecting vertices (i,j), increment m.set(i,j,m.get(i,j)+1)
    - Can be done during previous coordIndex traversal

How to construct from IndexedFaceSet ?

- 3) Link half-edges corresponding to regular edges
  - Have to set twinEdge for half-edges corresponding to m.get(i,j)==2
  - Use another symmetric sparse matrix data structure tw (or reuse m) initialized to NULL
  - Traverse half-edges again and save in the (i,j) position a pointer to the first corresponding half-edge (tw.get(i,j)==null)
  - When the second half-edge corresponding to the (i,j) position is visited (tw.get(i,j)!=null), and if the orientations are opposite, set the twinEdge fields of the current half-edge and the one stored in the (i,j) position of tw.
  - Resulting mesh has no singular edges (have been converted to boundary edges)

How to construct from IndexedFaceSet ?

- 4) Remove singular vertices
  - Use a partition data structure p with operations p.find(i), and p.join(i,j) to maintain a partition of the corners of the coordIndex array
  - Initialize to one singleton per corner
    - {{0},…{C-1}}
  - For each half-edge e with e.twinEdge!=null, join the corresponding corners
    - P.join(e.corner,e.twinEdge.nextEdge.corner)
  - Assign consecutive indices to the sets in the partition 0,…,P-1
  - Make new vertex coordinates array of length P
  - Set e.srcVertex to the partition number of e.corner