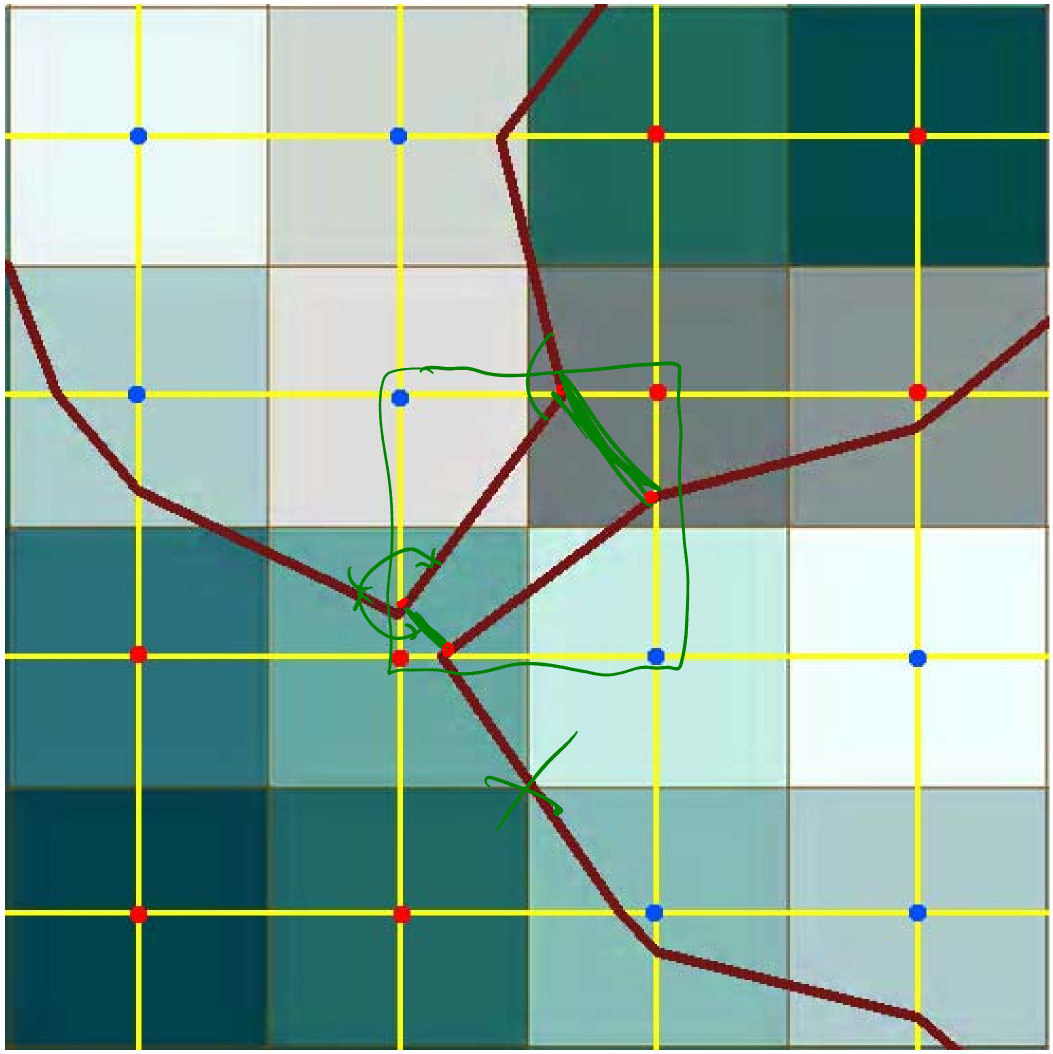
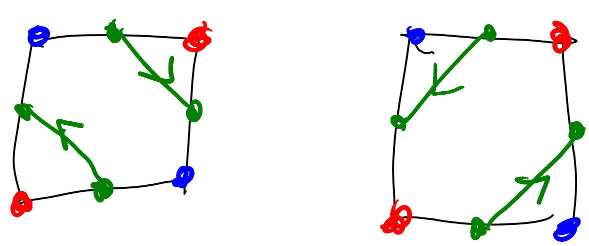
IsoCurves for Q-Meshes

Friday, February 01, 2008
2:00 PM





Baricentric Coordinates

Friday, February 01, 2008
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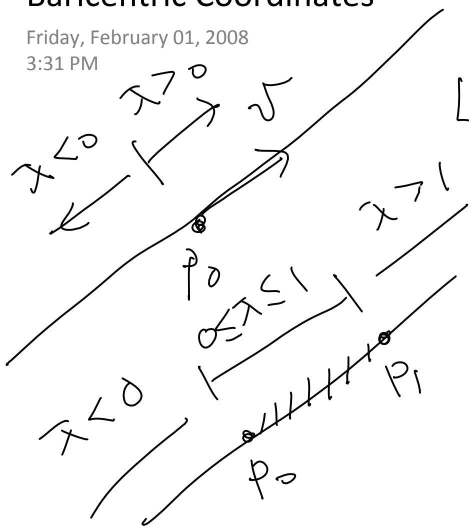


$$L = \{ p = p_0 + \lambda v \}$$

$$v = p_1 - p_0$$

$$p = p_0 + \lambda (p_1 - p_0) =$$

$$p = \underbrace{(1-\lambda)}_{b_0} p_0 + \underbrace{\lambda}_{b_1} p_1$$

$$\begin{cases} p = b_0 p_0 + b_1 p_1 \\ 1 = b_0 + b_1 \end{cases}$$

$$0 \le b_0, b_1$$

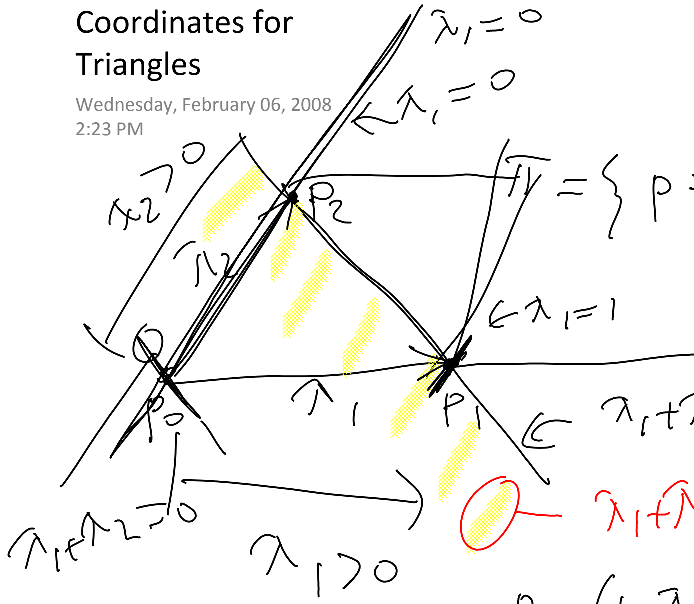
$$\begin{bmatrix} p \\ 1 \end{bmatrix} = \begin{bmatrix} p_0 & p_1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \end{bmatrix}$$

$$\boxed{p - p_0 = \lambda (p_1 - p_0)} \Rightarrow (p_1 - p_0)^t (p - p_0) = \lambda \| p_1 - p_0 \|^2$$

$$b_1 = \lambda = \frac{(p_1 - p_0)^t (p - p_0)}{\| p_1 - p_0 \|^2} \qquad b_0 = 1 - b_1$$

Barycentric Coordinates for Triangles

Wednesday, February 06, 2008
2:23 PM



$$\Pi = \{ p = p_0 + \lambda_1 (p_1 - p_0) + \lambda_2 (p_2 - p_0) \}$$

$$p = \underbrace{(1 - \lambda_1 - \lambda_2)}_{b_0} p_0 + \underbrace{\lambda_1}_{b_1} p_1 + \underbrace{\lambda_2}_{b_2} p_2$$

$$p = b_0 p_0 + b_1 p_1 + b_2 p_2$$

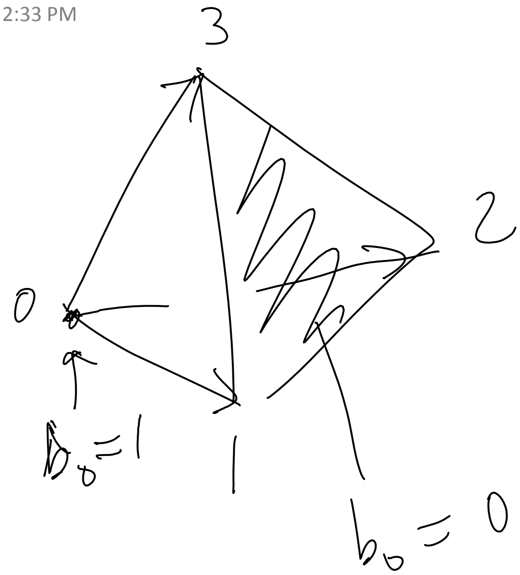
$$1 = b_0 + b_1 + b_2$$

$$0 \le b_0, b_1, b_2$$

$$n+1 \left[\begin{bmatrix} p \\ 1 \end{bmatrix} = \begin{bmatrix} p_0 & p_1 & p_2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix} \right.$$

Barycentric coordinates for tetrahedra

Wednesday, February 06, 2008
2:33 PM



$$p = b_0 p_0 + b_1 p_1 + b_2 p_2 + b_3 p_3$$

$$1 = b_0 + b_1 + b_2 + b_3$$

$$0 \le b_0, b_1, b_2, b_3$$

$$\begin{bmatrix} p \\ 1 \end{bmatrix} = \begin{bmatrix} p_0 & p_1 & p_2 & p_3 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Linear Interpolation

Wednesday, February 06, 2008
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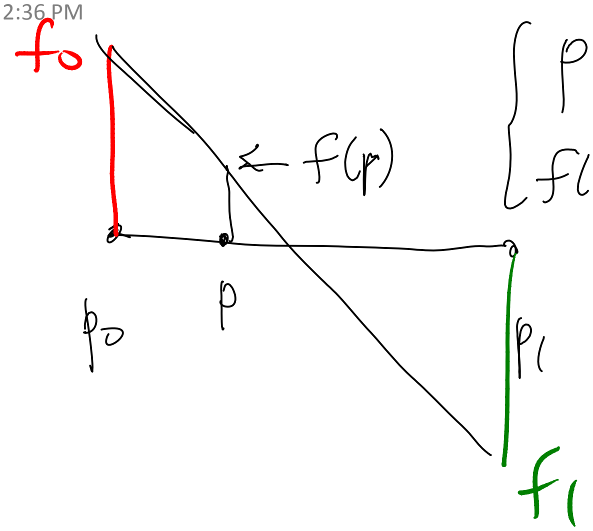

$$\begin{cases} p = p_0 + \lambda (p_1 - p_0) \\ f(p) = f_0 + \lambda (f_1 - f_0) \end{cases}$$

$$p = b_0 p_0 + b_1 f_1$$

$$f(p) = b_0 f_0 + b_1 f_1$$

Linear Interpolation

Wednesday, February 06, 2008
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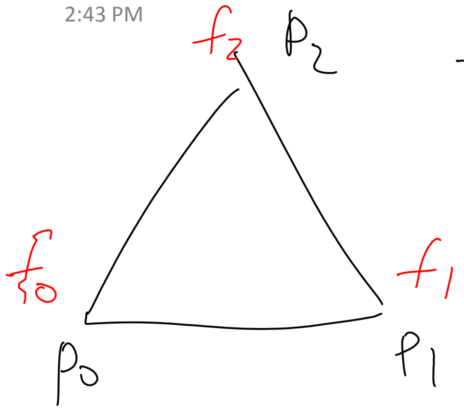

$$f(p) = b_0 f_0 + b_1 f_1 + b_2 f_2$$

$$4\left[\begin{bmatrix} p \\ 1 \end{bmatrix}\right] = \underbrace{\overbrace{\begin{bmatrix} p_0 & p_1 & p_2 \\ 1 & 1 & 1 \end{bmatrix}}^{3}}_{4\times 3} \begin{pmatrix} b_0 \\ b_1 \\ b_2 \end{pmatrix}$$

$$p - p_0 = \lambda_1 (p_1 - p_0) + \lambda_2 (p_2 - p_0)$$

$$[p - p_0] = \underbrace{\left[p_1 - p_0 \;\vdots\; p_2 - p_0 \right]}_{M} \underbrace{\begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix}}_{\lambda}$$

$$M^t (p - p_0) = \underbrace{M^t M}_{2\times 2} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$q = Ap + b \qquad A \quad 3\times 3$$

Q-mesh interpolation

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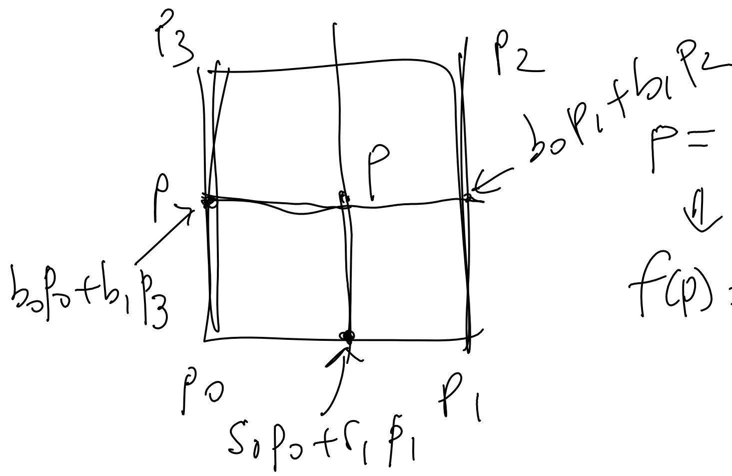
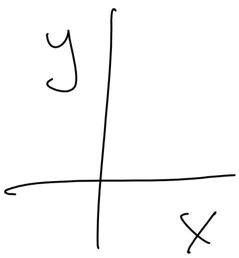


$$p = b_0 p_0 + b_1 p_1$$

$$\Downarrow$$

$$f(p) = b_0 f_0 + b_1 f_1$$



$$f(x,y) = f_{00} + f_{10} x + f_{01} y + f_{20} x^2 + f_{11} x y + f_{02} y^2$$

$$f(p) = f_0 \underbrace{\overbrace{s_0 b_0}^{\frac{1}{4}}}_{\substack{=1 \\ \text{for } p_0}} + f_1 \underbrace{s_1 b_0}_{\substack{=1 \\ \text{for } p_1}} + f_2 s_1 b_1 + f_3 s_0 b_1$$