Energy Function

Monday, February 25, 2008
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$$E(x) = \sum_{e=(i,j)} \|x_i - x_j\|^2$$

$$x^n \longmapsto x^{n+1} \quad \cdots \quad x^\infty$$

x^∞ is a min $\qquad i = 1, \ldots, N$

necessary $\quad \dfrac{\partial E}{\partial x_i}(x^\infty) = 0$

$$\frac{\partial E}{\partial x_i}\left(x_1^n, \ldots, x_{i-1}^n, x_i^{n+1}, x_{i+1}^n, \ldots, x_N^n\right) = 0$$

$$x_i^n + \delta_i x^n$$

Jacobi iteration = Laplacian Smoothing

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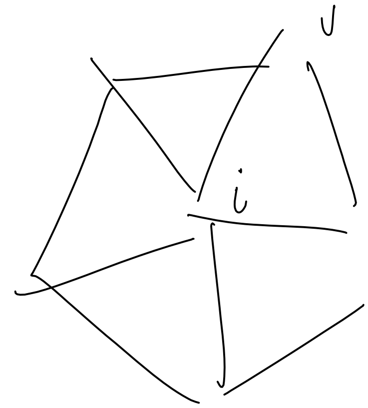
$$E(x) = \sum_{e=(h,j)} \|x_h - x_j\|^2$$



$$\frac{1}{2}\frac{\partial E}{\partial x_i}(x) = \sum_{j \in i^*} (x_i - x_j)$$

$$0 = \frac{1}{2}\frac{\partial E}{\partial x_i}\left(x_1^n, \ldots, \underbrace{x_i^{n+1}}_{x_i^n + \delta x_i^n}, \ldots\ x_N^n\right) = \sum_{j \in i^*} \left(x_i^n + \delta x_i^n - x_j^n\right)$$

$$\underbrace{\sum_{j \in i^*} \delta x_i^n} = \sum_{j \in i^*} (x_j^n - x_i^n) \qquad i = 1 \ldots N$$

$$|i^*|\, \delta x_i^n = \sum_{j \in i^*} (x_j^n - x_i^n)$$

$$\boxed{\delta x_i^n = \frac{1}{|i^*|} \sum_{j \in i^*} (x_j^n - x_i^n)}$$