Subdivision Curves

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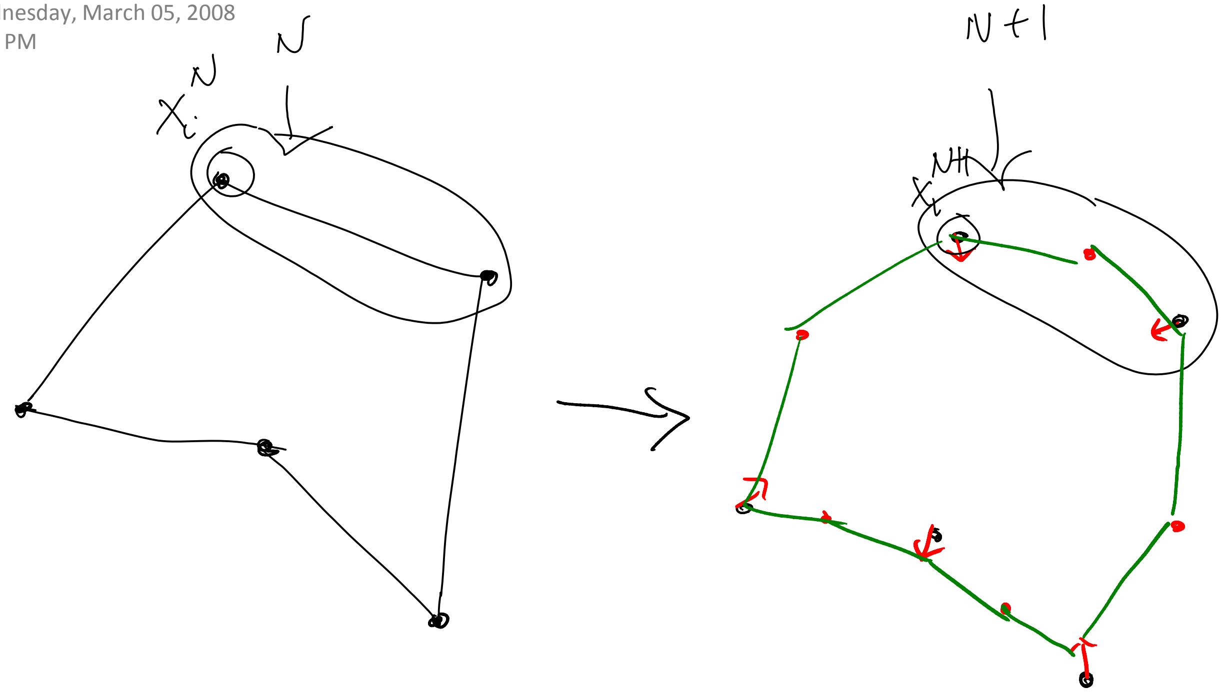

$$x_n \to x \qquad |x_n - x| \to 0$$

Subdivision of 1D signals

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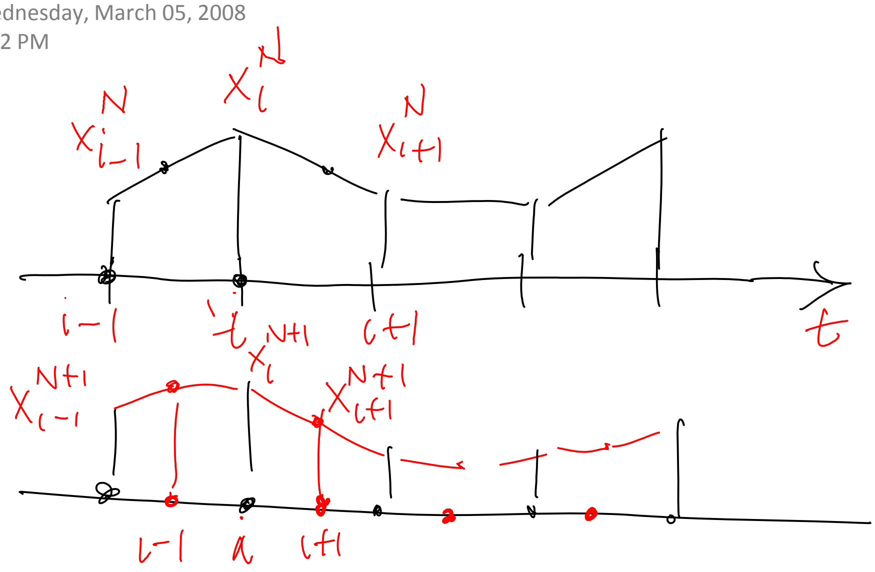
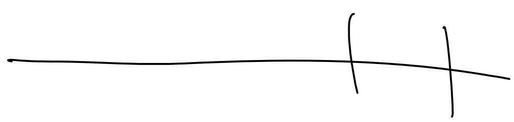


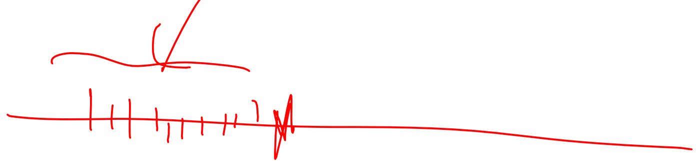
$$\begin{bmatrix} x_{i-1}^{N+1} \\ x_i^{N+1} \\ x_{i+1}^{N+1} \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 1 & 0 \\ 0 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} x_{i-1}^{N} \\ x_i^{N} \\ x_{i+1}^{N} \end{bmatrix}$$



$\frac{0}{4}$ $\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$ $\frac{4}{4}$

N

$\frac{j}{2^N}$ $\quad 0 \leq j \leq 2^N$



$t_j \longrightarrow t$

$f(t_j)$

Matrix Formulation

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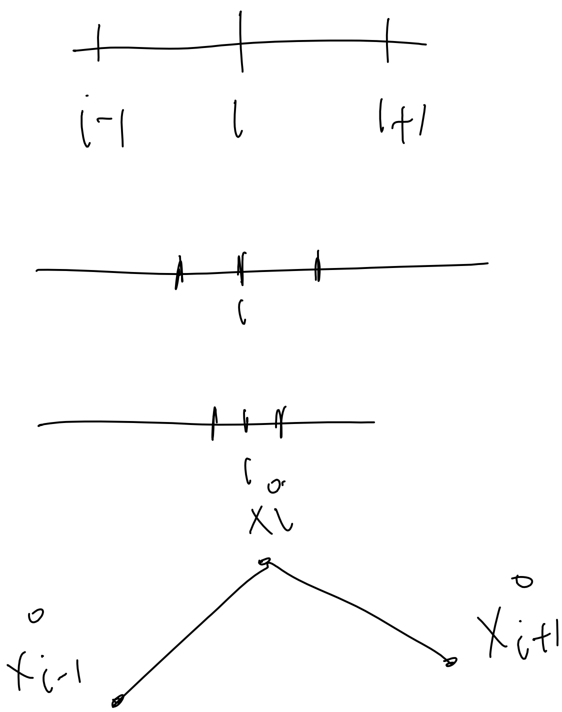
$$\begin{bmatrix} x_{i-1}^{N+1} \\ x_i^{N+1} \\ x_{i+1}^{N+1} \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 1 & 0 \\ 0 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} x_{i-1}^{N} \\ x_i^{N} \\ x_{i+1}^{N} \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} x_{i-1}^{N+1} \\ x_i^{N+1} \\ x_{i+1}^{N+1} \end{bmatrix}}_{\underline{X}_i^{N+1}} = \underbrace{\begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/8 & 3/4 & 1/8 \\ 0 & 1/2 & 1/2 \end{bmatrix}}_{M} \underbrace{\begin{bmatrix} x_{i-1}^{N} \\ x_i^{N} \\ x_{i+1}^{N} \end{bmatrix}}_{\underline{X}_i^{N}}$$



$$\underline{X}_i^{N+2} = M^2 \underline{X}_i^N$$

$$\Downarrow$$

$$\underline{X}_i^{N} = M^N \underline{X}_i^0$$

$$x_i^N = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} M^N \underline{X}_i^0$$

M : normal matrix

$$AMA^{-1} \text{ is symmetric} \Updownarrow AA^t = A^tA$$

$$M \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \overset{\lambda_1 = 1}{\downarrow} \underbrace{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}_{e_1}$$

$$1 = \lambda_1 > \lambda_2 \geq \lambda_3 > 0$$

$$e_1 \quad e_2 \quad e_3$$

$$M^N \longrightarrow \begin{bmatrix} \frac{1}{6} & \frac{2}{3} & \frac{1}{6} \\ \frac{1}{6} & \frac{2}{3} & \frac{1}{6} \\ \frac{1}{6} & \frac{2}{3} & \frac{1}{6} \end{bmatrix} = M^\infty$$

$$X_c^\infty = \lim_{N \to \infty} X_i^N = \underbrace{\begin{bmatrix} 0 & 1 & 0 \end{bmatrix} M^\infty}_{\begin{bmatrix} \frac{1}{6} & \frac{2}{3} & \frac{1}{6} \end{bmatrix}} \underline{X}_i^0 \quad \begin{bmatrix} X_{i-1}^0 \\ X_c^0 \\ X_{i+1}^0 \end{bmatrix}$$

Eigenanalysis

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$$M \quad \left.\begin{matrix} \lambda_1 & \lambda_2 & \lambda_3 \\ e_1 & e_2 & e_3 \end{matrix}\right) \qquad M e_j = \lambda_j e_j$$

$$\underline{X}_i^0 = a_i e_1 + b_i e_2 + c_i e_3$$

$$\underline{X}_i^1 = M \cancel{\underline{X}}_i^0 = a_i \underbrace{M e_1}_{\lambda_1 e_1} + b_i \underbrace{M e_2}_{\lambda_2 e_2} + c_i \underbrace{M e_3}_{\lambda_3 e_3}$$

$$\underbrace{\underline{X}_i^N}_{=} = a_i \lambda_1^N e_1 + b_i \lambda_2^N e_2 + c_i \lambda_3^N e_3$$

$$\lambda_1^N = 1 \qquad \lambda_2^N \to 0 \qquad \lambda_3^N \to 0$$

$$\begin{pmatrix} X_{i-1}^N \\ X_i^N \\ X_{i+1}^N \end{pmatrix} \qquad |X_{i+1}^N - X_i^N| \to 0$$

$$\lim_{N \to \infty} \frac{X_{i+1}^N - X_i^N}{1/2^N} = \text{right derivative}$$

$$\lim_{N \to \infty} \frac{X_i^N - X_{i-1}^N}{1/2^N} = \text{left derivative}$$

$$\underbrace{2^N \left(X_{i+1}^N - 2X_i^N + X_{i-1}^N \right)}_{} \to 0$$

$$2^N \begin{bmatrix} 1 & -2 & 1 \end{bmatrix} \underbrace{\underline{X}_i^N} \to 0$$

$$2^N[1\ -2\ 1]\left(a_i\lambda_1^N e_1 + b_i\lambda_2^N e_2 + c_i\lambda_3^N e_3\right) \to 0$$

$$b_i\underbrace{\left(2^N\lambda_2^N\right)}_{\to 0}[1\ -2\ 1]e_2 + d_i\underbrace{\left(2^N\lambda_3^N\right)}_{\to 0}[1\ -2\ 1]e_3$$

necessary that

$$0 < |\lambda_2|, |\lambda_3| < 2$$

How to extend analysis to meshes

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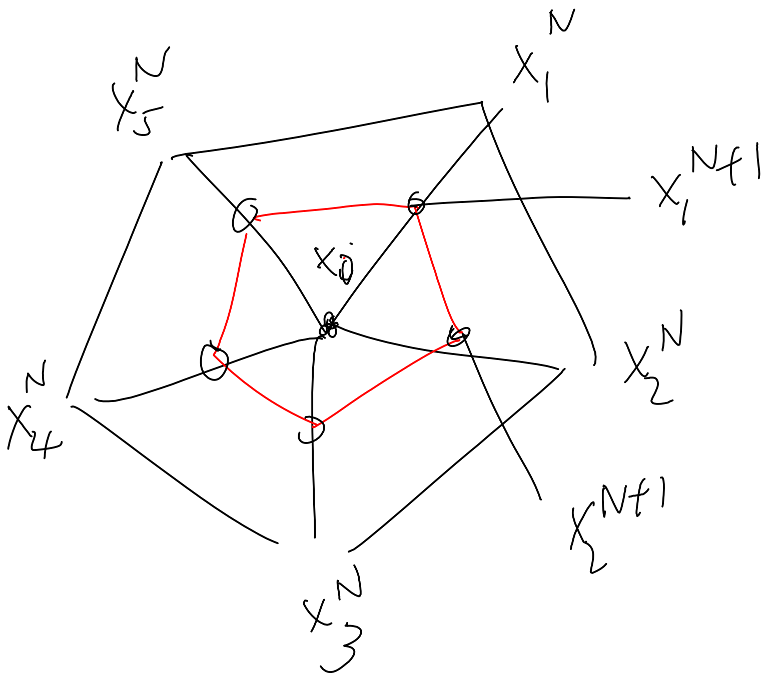

$$\begin{bmatrix} x_0^N \\ x_1^N \\ \vdots \\ x_K^N \end{bmatrix} \leftarrow \underline{X}^N$$

$$\boxed{S}\ \boxed{\underline{X}^N} = \underline{X}^{N+1}$$