

## Assignment 3 : Mesh Optimization

Friday, March 07, 2008

12:56 PM

- Unified approach to Smoothing with hard and soft constraints
- Edge collapse simplification
- Adaptive triangle mesh subdivision
- Optimization by Edge flipping

# Unified approach to Smoothing with hard and soft constraints

Friday, March 07, 2008

1:00 PM

$$E(x) = \sum_{e=(ij)} \|x_i - x_j\|^2 + \lambda \sum_{i \in I} \|x_i - x_i^0\|^2$$

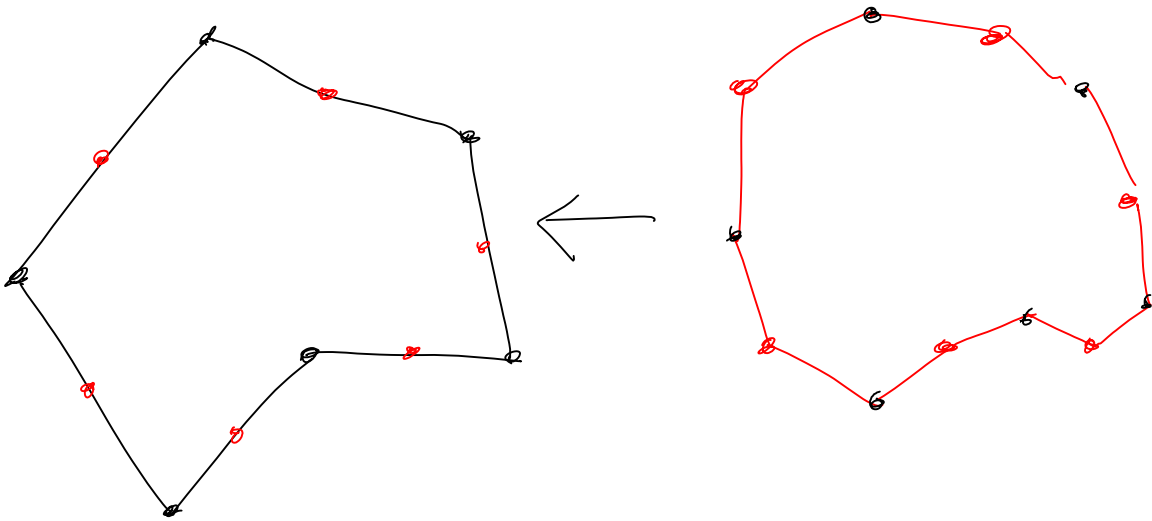
$$\downarrow$$
$$\Delta x_i$$

$$\frac{\partial E}{\partial x_i} (x_1, \dots, x_i + \delta x_i, \dots, x_N) = 0 \rightarrow$$

$$\delta x_i = \Phi_i(x)$$

# First Order Energy

Friday, March 07, 2008  
2:16 PM



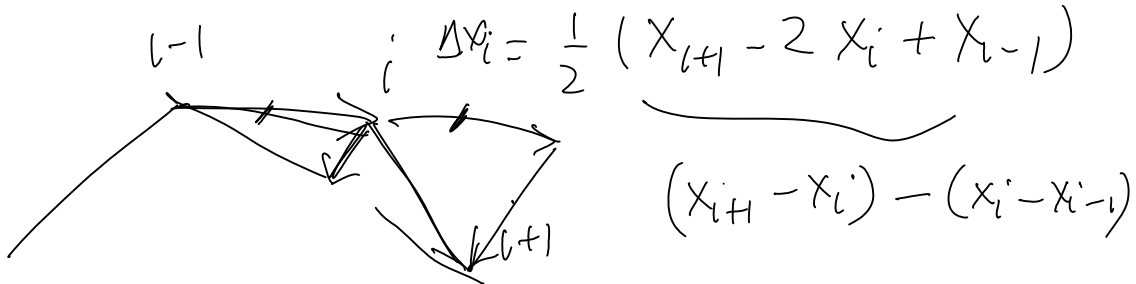
$$E(x) = \sum_{(i,j)} \|x_i - x_j\|^2$$

# Second Order Energy

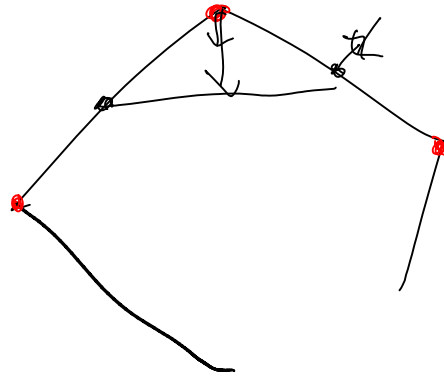
Friday, March 07, 2008  
2:19 PM

A symmetric  $\geq 0$

$$E(x) = \sum_i \|\Delta x_i\|^2 = x^t A x$$



$$E(\tilde{x}) = \tilde{\tau}^2 E(x)$$



$$E(x) = x^t A x + 2b^t x + c$$

$$0 = \frac{1}{2} \frac{\partial E}{\partial x} = Ax + b \Rightarrow$$

$$\hat{x} = -A^{-1} b$$

# Matrix derivatives

Friday, March 07, 2008  
2:29 PM

$$\frac{\partial}{\partial x_j} (x^t A x) = \dots$$

$J = \begin{pmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix}^t \quad A = \begin{pmatrix} x_1 \\ \vdots \\ x_N \end{pmatrix}$

$$\underbrace{\frac{\partial}{\partial x_j} (x)^t}_{\text{row vector}} \cdot A x + x^t \underbrace{\frac{\partial}{\partial x_j} (A x)}_{\text{column vector}}$$

$$\frac{\partial}{\partial x_j} \begin{pmatrix} x_1 \\ \vdots \\ x_N \end{pmatrix} \quad A \frac{\partial}{\partial x_j} (x)$$

$$\begin{pmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix}^t \quad \left( \begin{array}{|c|} \hline \square \\ \hline \end{array} \right)$$

$$\frac{\partial}{\partial x} (x^t A x) = 2 A x$$

$$\frac{\partial}{\partial x} (b^t x) = \begin{pmatrix} \frac{\partial}{\partial x_1} (b^t x) \\ \vdots \\ \frac{\partial}{\partial x_N} (b^t x) \end{pmatrix} = b$$

# Jacobi iteration

Friday, March 07, 2008  
2:36 PM

$$E(x) = x^t A x + 2b^t x + C$$

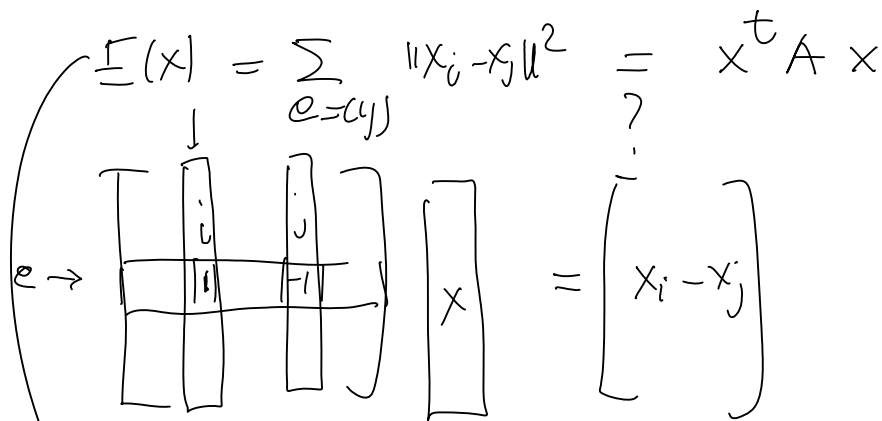
$$\frac{1}{2} \frac{\partial E}{\partial x}(x) = Ax + b \quad D = \text{diag}(A)$$

$$\frac{\partial E}{\partial x_j}(x) = (a_{j1} \dots a_{jN})x + b_j$$

$$0 = \frac{\partial E}{\partial x_j}(x_1, \dots, x_j + \delta x_j, \dots, x_N) = (a_{j1}, \dots, a_{jN})x + b_j + a_{jj} \delta x_j$$

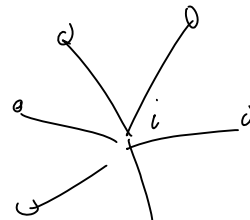
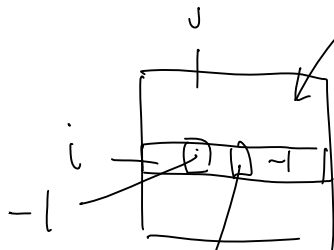
$$0 = Ax + b + D \delta x \Rightarrow$$

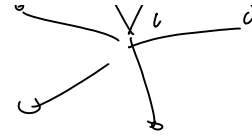
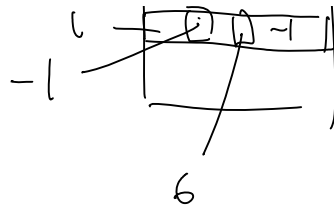
$$\boxed{\delta x = -D^{-1}(Ax + b)}$$

$$\Gamma(x) = \sum_{e=(ij)} \|x_i - x_j\|^2 = x^t A x$$


The diagram illustrates the relationship between the graph Laplacian and the quadratic form. It shows a graph with nodes  $i$  and  $j$  connected by an edge  $e$ . A vector  $x$  is shown with components  $x_i$  and  $x_j$ . The difference  $x_i - x_j$  is shown in a vertical vector. The Laplacian  $L$  is shown as a matrix with  $-1$  on the diagonal and  $1$  on the off-diagonal for the edge  $e$ .

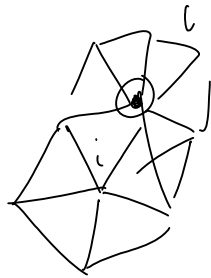
$$\|Lx\|^2 = x^t \begin{pmatrix} A \\ L \\ L \end{pmatrix} x$$





$$\delta X = -D^{-1} A X = \begin{pmatrix} \Delta x_i \end{pmatrix}$$

$$E(X) = \sum \|\Delta x_i\|^2 \rightarrow \delta^2 x_i ?$$



$$\delta^2 x_i = \frac{1}{1+6\epsilon} \sum_{j \in i^*} \Delta x_i - \Delta x_j$$