Assignment 3 : Mesh Optimization

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-Unified approach to Smoothing with hard and soft constraints
-Edge collapse simplification
-Adaptive triangle mesh subdivision
-Optimization by Edge flipping

Unified approach to Smoothing with hard and soft constraints

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$$E(x) = \sum_{e=(ij)} \|x_i - x_j\|^2 + \xi \sum_{i \in I} \|x_i - x_i^0\|^2$$

$$\downarrow$$

$$\Delta x_i$$

$$\frac{\partial E}{\partial x_i}(x_1 \dots x_i + \delta x_i, \dots x_N) = 0 \rightarrow$$

$$\delta x_i = \Phi_i(x)$$

First Order Energy

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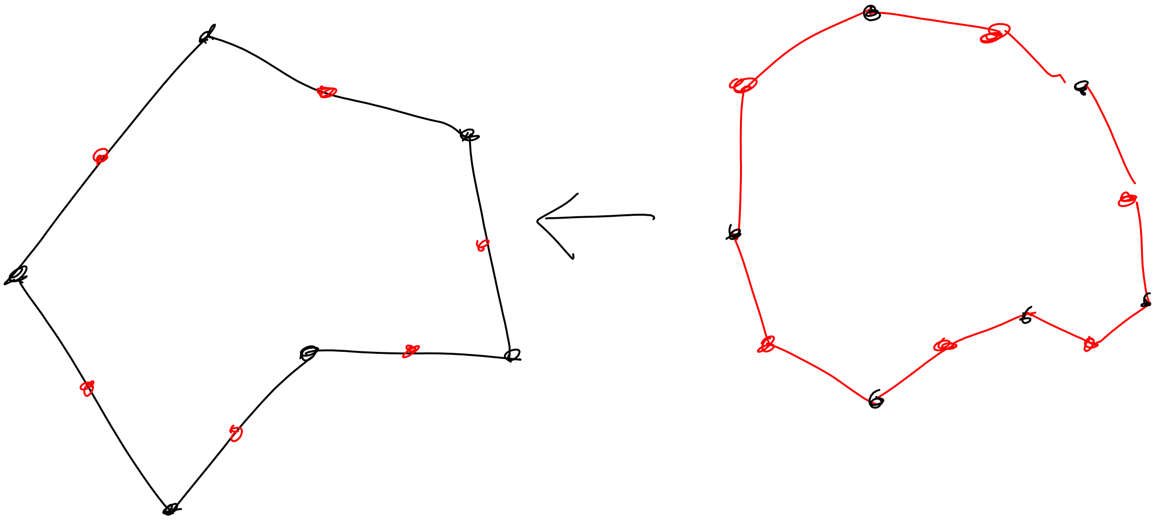

$$E(x) = \sum_{(i,j)} \|x_i - x_j\|^2$$

Second Order Energy

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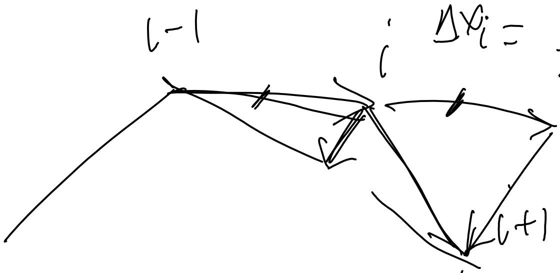
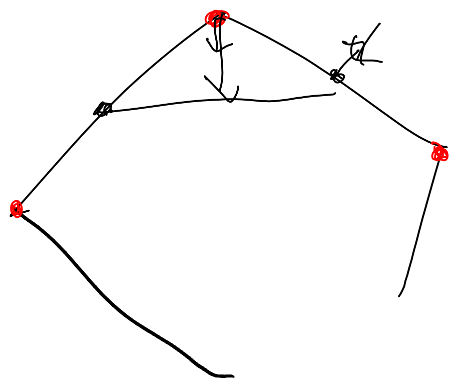
A symmetric ≥ 0

$$E(x) = \sum_i \|\Delta x_i\|^2 = x^t A x$$



$$\Delta x_i = \frac{1}{2}\underbrace{(x_{i+1} - 2x_i + x_{i-1})}_{(x_{i+1}-x_i)-(x_i-x_{i-1})}$$

$$E(\xi x) = \xi^2 E(x)$$



$$E(x) = x^t A x + 2b^t x + c$$

$$0 = \frac{1}{2}\frac{\partial E}{\partial x} = Ax + b \Rightarrow \boxed{\hat{x} = -A^{-1} b}$$

Matrix derivatives

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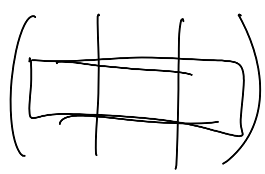
$$\frac{\partial}{\partial x_j}\left(x^t A x\right) = \cancel{x}$$

$$\begin{array}{c c} & \\ j & \end{array}\begin{pmatrix}0\\ \vdots\\ 1\\ \vdots\\ 0\end{pmatrix}^t A \begin{pmatrix}x_1\\ \vdots\\ x_N\end{pmatrix}$$

$$\underbrace{\frac{\partial}{\partial x_j}(x)}_{\frac{\partial}{\partial x_j}\begin{pmatrix}x_1\\ \vdots\\ x_N\end{pmatrix}}{}^t \cdot Ax + x^t \underbrace{\frac{\partial}{\partial x_j}(Ax)}_{A\frac{\partial}{\partial x_j}(x)}$$

$$\begin{pmatrix}0\\ 1\\ 0\end{pmatrix}^t$$



$$\frac{\partial}{\partial x}\left(x^t A x\right) = 2Ax$$

$$\frac{\partial}{\partial x}\left(b^t x\right) = \begin{pmatrix}\frac{\partial}{\partial x_1}(b^t x)\\ \vdots\\ \frac{\partial}{\partial x_N}(b^t x)\end{pmatrix} = b$$

Jacobi iteration

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$$E(x) = x^t A x + 2b^t x + C$$

$$\frac{1}{2}\frac{\partial E}{\partial x}(x) = Ax + b \qquad D = diag(A)$$

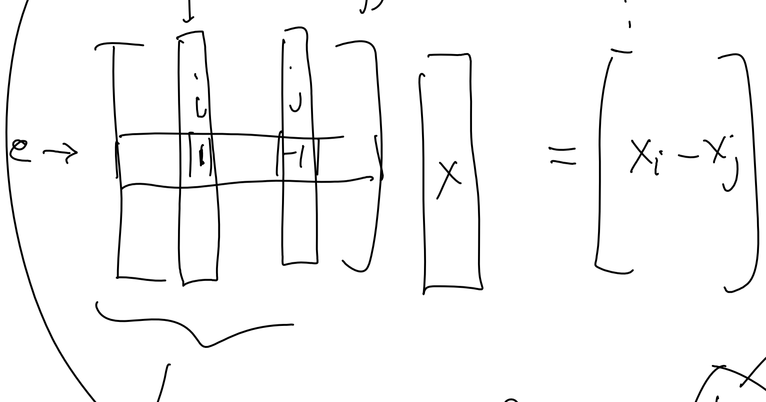
$$\frac{\partial E}{\partial x_j}(x) = (a_{j1} \dots a_{jN})\,x + b_j$$

$$0 = \frac{\partial E}{\partial x_j}(x_1 \dots x_j + \delta x_j, \dots, x_N) = (a_{j1}, \dots, a_{jN})\,x + b_j + a_{jj}\,\delta x_j$$

$$0 = Ax + b + D\,\delta x \Rightarrow$$

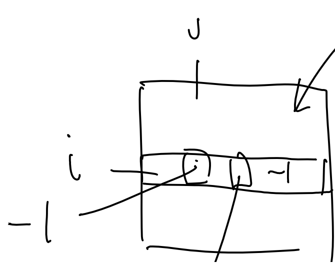
$$\boxed{\delta x = -D^{-1}(Ax + b)}$$

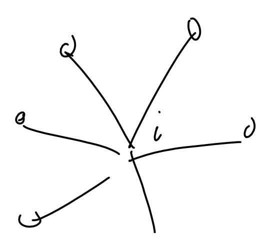
$$E(x) = \sum_{e=(i,j)} \|x_i - x_j\|^2 \overset{?}{=} x^t A x$$

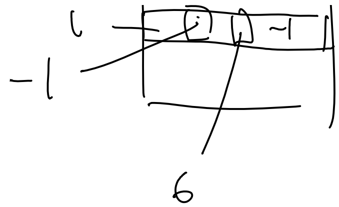


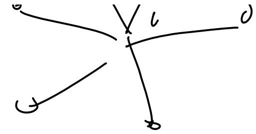
$$\|Lx\|^2 = x^t L^t L\, x$$

A



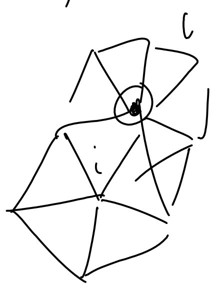






$$\delta x = -D^{-1} A x = \left(\Delta x_i\right)$$

$$E(x) = \sum \|\Delta x_i\|^2 \longrightarrow \delta^2 x_i \ ?$$



$$\delta^2 x_i = \frac{-1}{1+|i^*|} \sum_{j \in i^*} \Delta x_i - \Delta x_j$$