Curves

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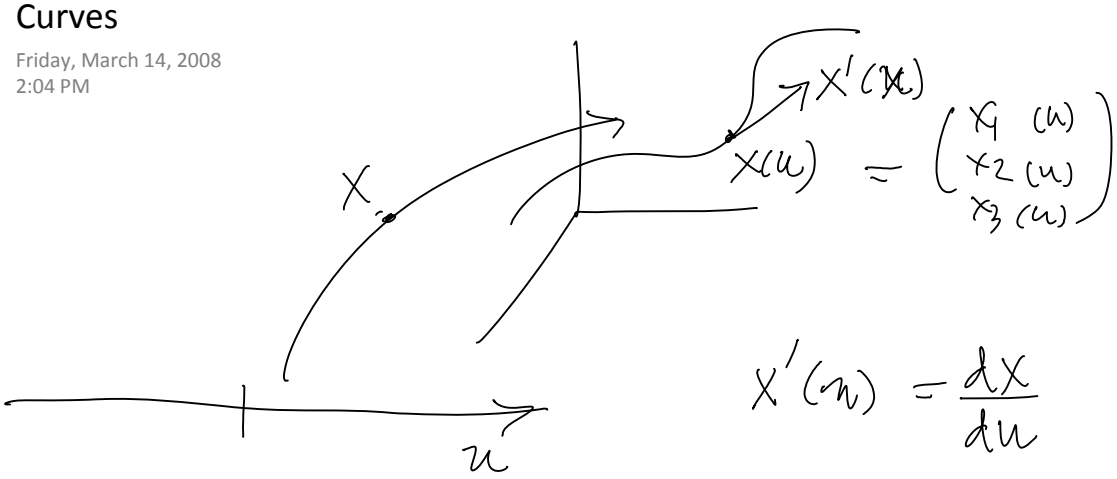
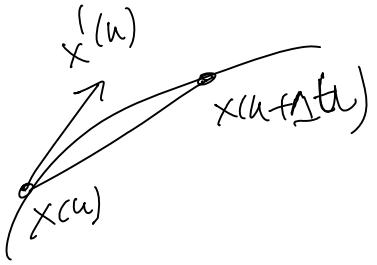


$$x'(u) = \frac{dx}{du}$$

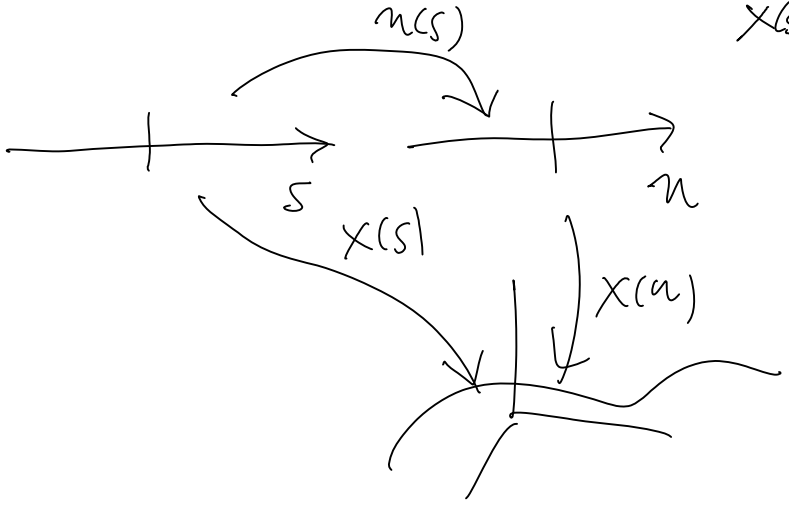


$$x'(u) = \lim_{\Delta u \to 0} \frac{x(u+\Delta u) - x(u)}{\Delta u}$$

$$x''(u)$$

$$\| x'(u) \| = x'(u)^t x'(u)$$

$$x(s) = x(u(s))$$

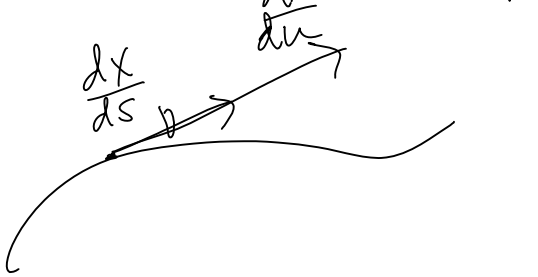


$$\underbrace{\frac{dx}{ds}}_{\text{vector}} = \underbrace{\frac{dx}{du}(u(s))}_{\text{vector}} \underbrace{\frac{du}{ds}}_{\text{scalar}} \neq 0$$

$\frac{dx}{du}$

$\frac{dx}{ds}$

vector vector $\frac{dX}{du}$ scalar



$$T(s) = \frac{X'(s)}{\|X'(s)\|} \qquad \|T\| = 1$$

arc len

Arc Length Parameterization

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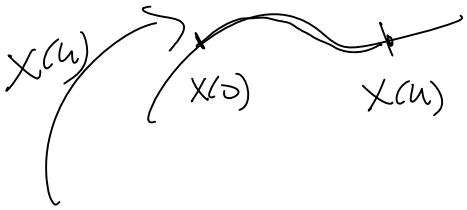
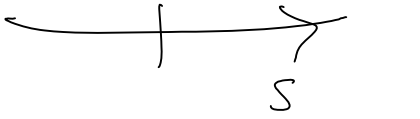
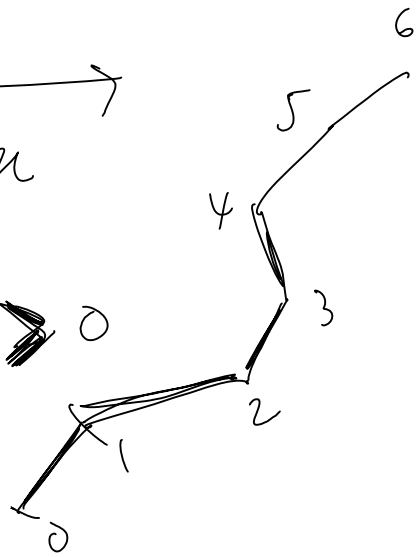
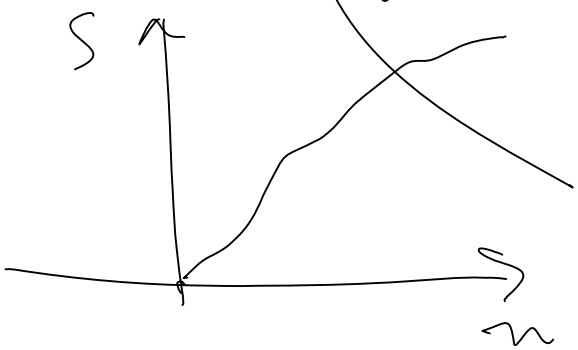


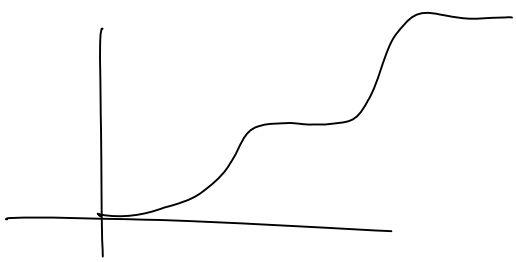




$$s(u) = \int_0^u \|x'(\tau)\| \, d\tau \geq 0$$



$$\frac{ds}{du}(u) = \|x'(u)\| > 0$$



$u(s)$ ← inverse of $s(u)$

$$X(s) = X(u(s))$$

$$\Downarrow$$

$$\|x'(s)\| = 1$$

Frenet Frame 1

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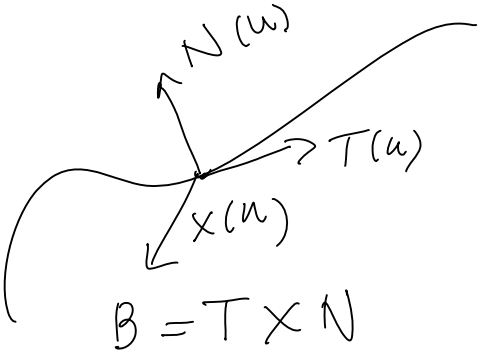

$B = T \times N$

$$T(u) = \frac{x(u)}{\|x'(u)\|}$$

$\|T\|^2 = 1$ $\quad \|T'(u)\|$

$$T^t T = 1 \Rightarrow \frac{d}{du}(T^t T) = 0$$

$$\| \qquad 2\, T^t T'$$

$\kappa = \|T'\|$ normal curvature $\neq 0$

$N = \frac{T'}{\kappa}$ $\qquad \boxed{T' = \kappa N}$

Frenet Frame 2

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$\|x'(u)\|\, T(u) = x(u)$

binormal

$T(u)$, $N(u)$ $\quad B = T \times N \quad [T, N\, B] = R$

$$\begin{cases} T' = 0T + \kappa N + 0B \\ N' = -\kappa T + 0N + \tau B \\ B' = 0T - \tau N + 0B \end{cases}$$

$N^t T = 0 \Rightarrow (N')^t T + N^t T' = 0$ ($N^t T'$: κ)

$N' = (T^t N')T + (N^t N')N + (B^t N')B$

$(T^t N') = -\kappa$, $(N^t N') = 0$

$N^t N = 1 \Rightarrow 2N^t N' = 0$

$N' + \kappa T \perp T, N$

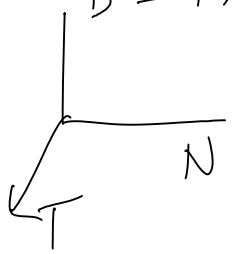
$B^t(N' + \kappa T) = \tau$ torsion

$B = T \times N \Rightarrow B' = T' \times N + T \times N'$

$T' \times N = \kappa N$

$-\kappa T + \tau B =$

$$B = T \times N$$



$$T \times T = 0$$

$$T \times B = -N$$

$$T \times N = B$$

Frenet Frame

$$\begin{pmatrix} T \\ N \\ B \end{pmatrix}' = \begin{pmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{pmatrix} \begin{pmatrix} T \\ N \\ B \end{pmatrix}$$

Skew-symmetric

$$K^t + K = 0$$

Surfaces

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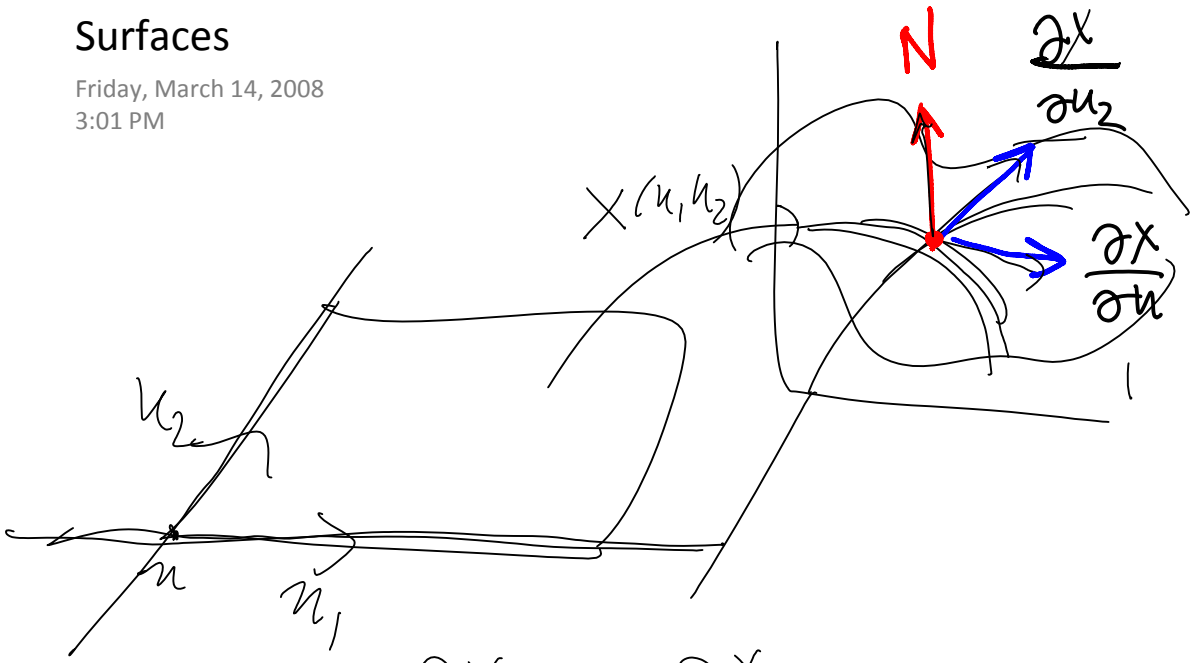

$$N = \frac{\frac{\partial X}{\partial u_1} \times \frac{\partial X}{\partial u_2}}{\left\| \frac{\partial X}{\partial u_1} \times \frac{\partial X}{\partial u_2} \right\|}$$