Curvature and for Polygonal Curves

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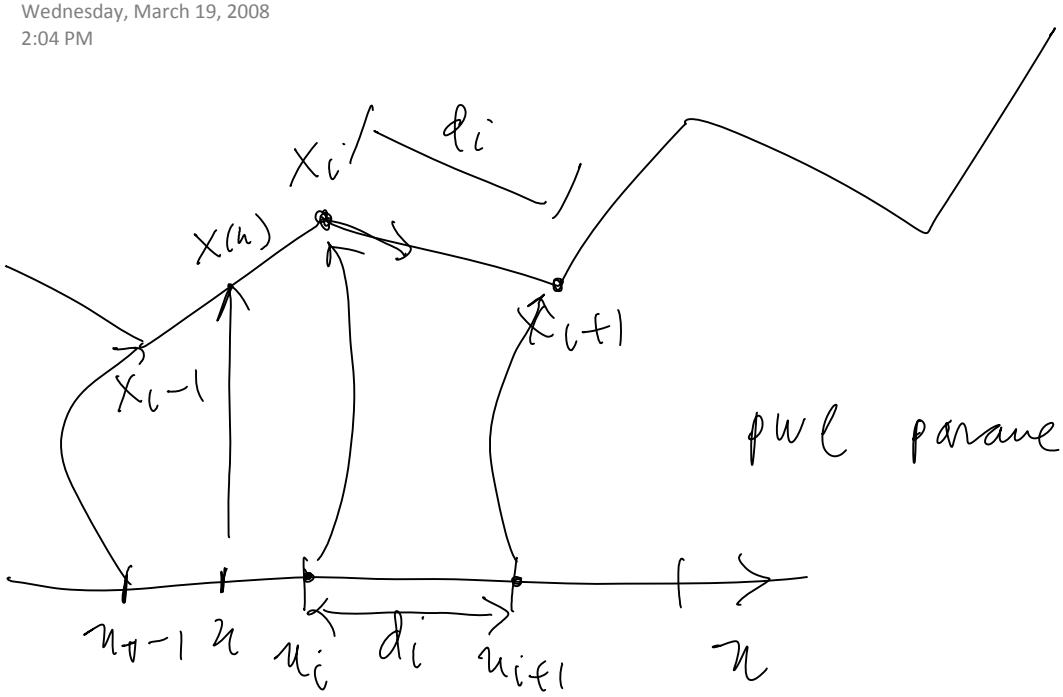


pwl parameterization

$$x(u) = \begin{cases} \\ \end{cases} \qquad x(u_i) = x_i$$

$u_{i-1} \le u \le u_i$

$u = (1-t)\,u_{i-1} + t\,u_i \qquad 0 \le t \le 1$

$$x(u) = (1-t)\,x_{i-1} + t\,x_i$$

arc length parameterization

$u_{i+1} - u_i = \| x_{i+1} - x_i \| = d_i$

$$x'(u) = \begin{cases} \dfrac{x_{i+1} - x_i}{d_i} \\ \end{cases} \qquad u_i \le u \le u_{i+1}$$

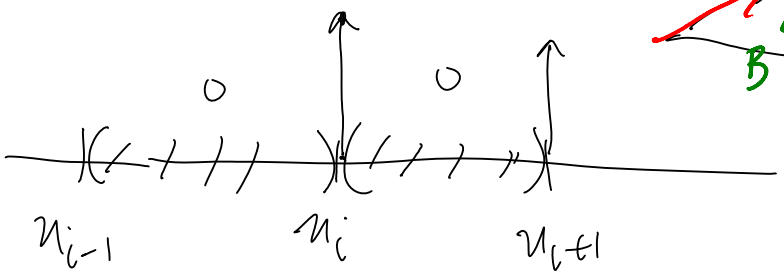
$$\frac{x_{i+1} - x_i}{\| x_{i+1} - x_i \|}$$

pwc

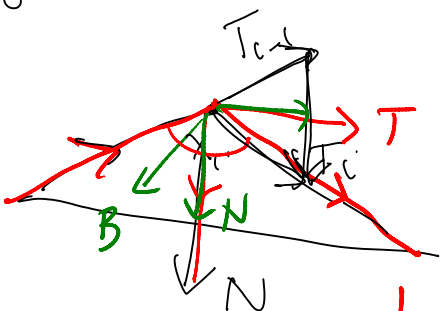
$$T = \frac{x'}{\quad} \; - x'$$

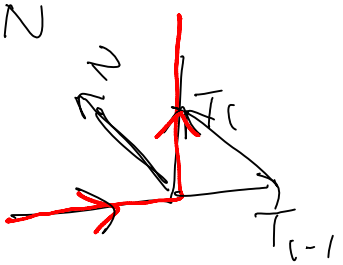
$\|X'\|$ ← arc length param

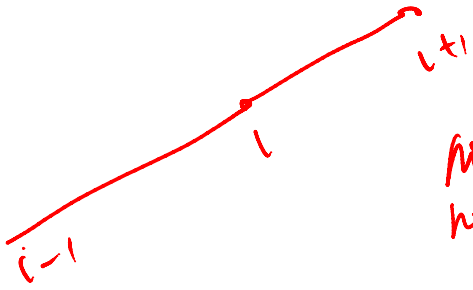
$T' = \kappa N$



$B = T \times N$







normal
not unique

Torsion of polygonal curves

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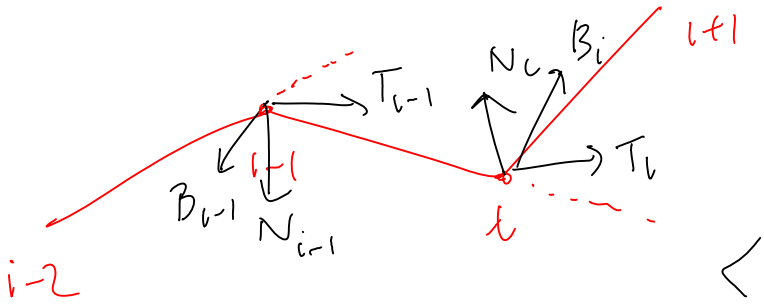
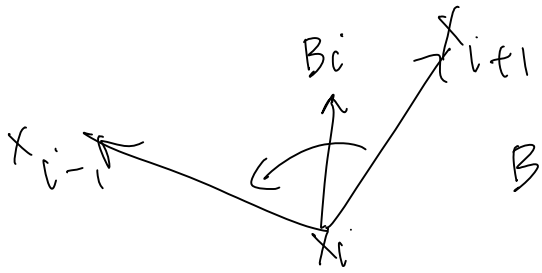


$$\langle X_{i+1} - X_i,\ X_{i-1} - X_i \rangle = \langle T_i,\ N_i \rangle$$

$$N' = -\kappa T + \tau B$$

$$N_i - N_{i-1}$$



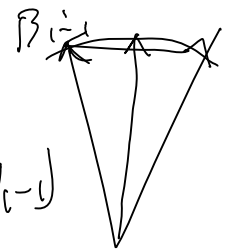
$$B_i = T_i \times N_i$$

$$B_i = \frac{(X_{i+1} - X_i) \times (X_{i-1} - X_i)}{\| (X_{i+1} - X_i) \times (X_{i-1} - X_i) \|}$$

$$\tau = B^t N'$$

$$\simeq \frac{1}{2}(B_i + B_{i-1})^t (N_i - N_{i-1})$$

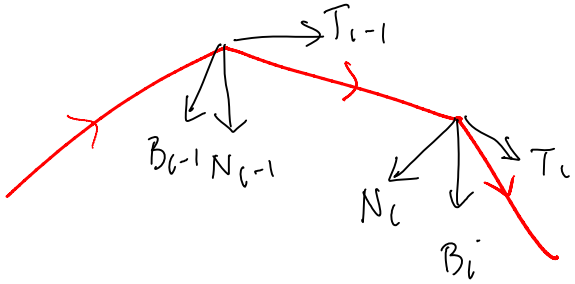
?



?

ideally

$$\tau = \| B_i - B_{i-1} \|$$



Degrees of freedom

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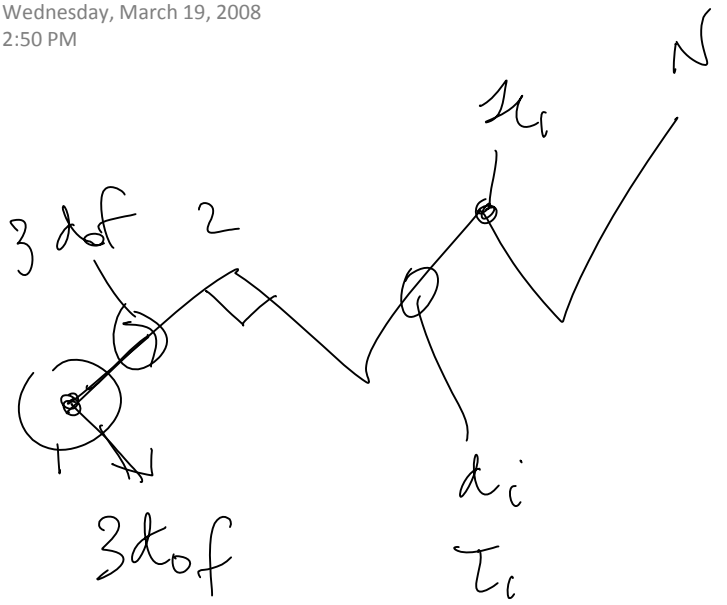
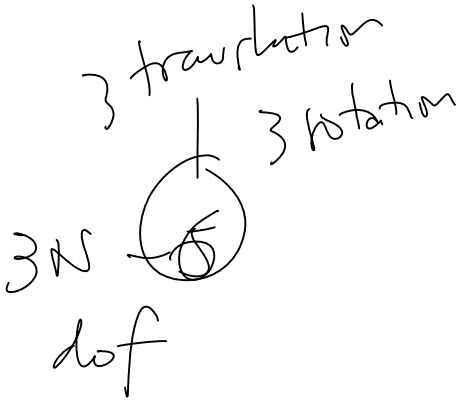
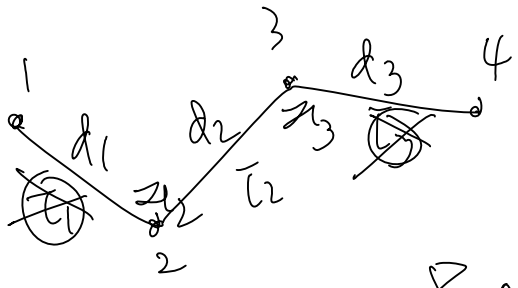




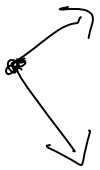
$(N-2)$ ... θ_i's
$(N-1)$ d_i's
$(N-3)$ α_i's

$3N-6$

$N = 4$



8 parameters
+6
14




I
N