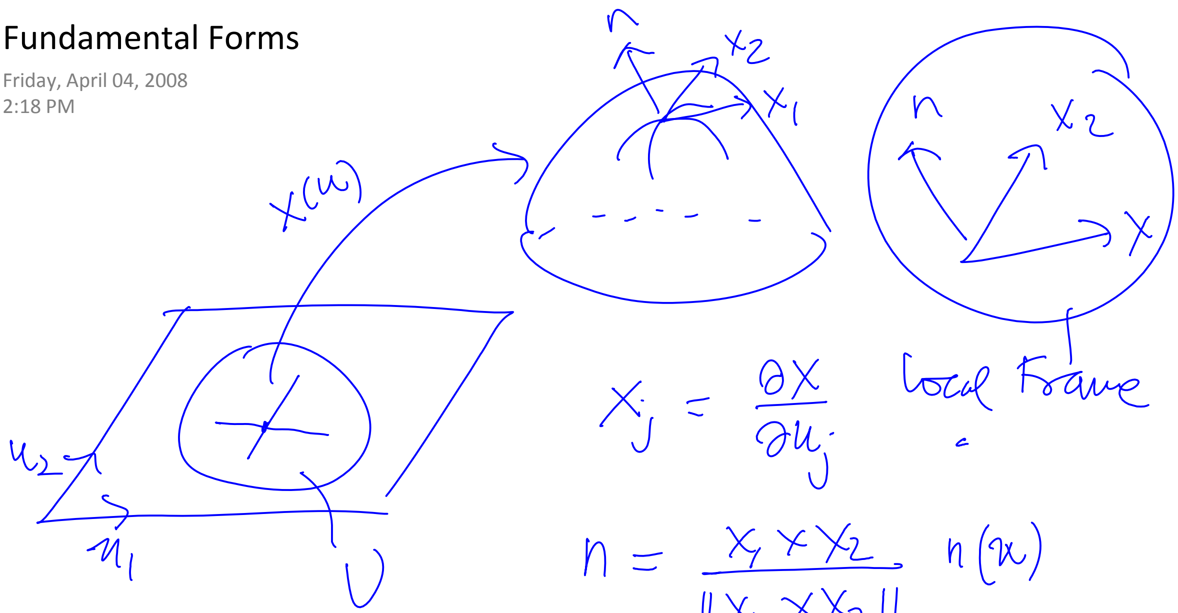
Fundamental Forms

Friday, April 04, 2008
2:18 PM



$x_j = \frac{\partial x}{\partial u_j}$

$n = \frac{x_1 \times x_2}{\|x_1 \times x_2\|}$ $\quad n(x)$

I $\quad g_{ij} = x_i^t x_j$

$g_{12} = g_{21}$

$G = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$ First Fundam form

$n_i^t x_j = n_j^t x_i$

$\frac{\partial^2 x}{\partial u_i \partial u_j}$

$\underline{n_j} = \frac{\partial n}{\partial u_j}$ $\qquad n^t x_j = 0$

$0 = \frac{\partial}{\partial u_i}(n^t x_j) = \boxed{n_i^t x_j + n^t x_{ij}}$

$x_{ij} = x_{ji}$

$n_i = \Box\, x_1 + \Box\, x_2 + (0)\, n$

$1 = \|n\|^2 = n^t n \Rightarrow 0 = \frac{\partial}{\partial u_i}(n^t n) = 2 n^t n_i$

$x_j^t n_i = \overset{?}{\Box}\, \underbrace{x_j^t x_1}_{g_{j1}} + \overset{?}{\Box}\, \underbrace{x_j^t x_2}_{g_{j2}}$

$\cancel{n^t x_i}$ $\quad g_{j1} \quad g_{j2}$

2nd fundamental form

$$x_{ij} = \Gamma_{ij}^1 x_1 + \Gamma_{ij}^2 x_2 + h_{ij} n$$

Christofel symbols of 2nd king

$$H = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$$

$$n^t x_{ij} = h_{ij} = -n_i^t x_j = -n_j^t x_i$$

Vector Fields

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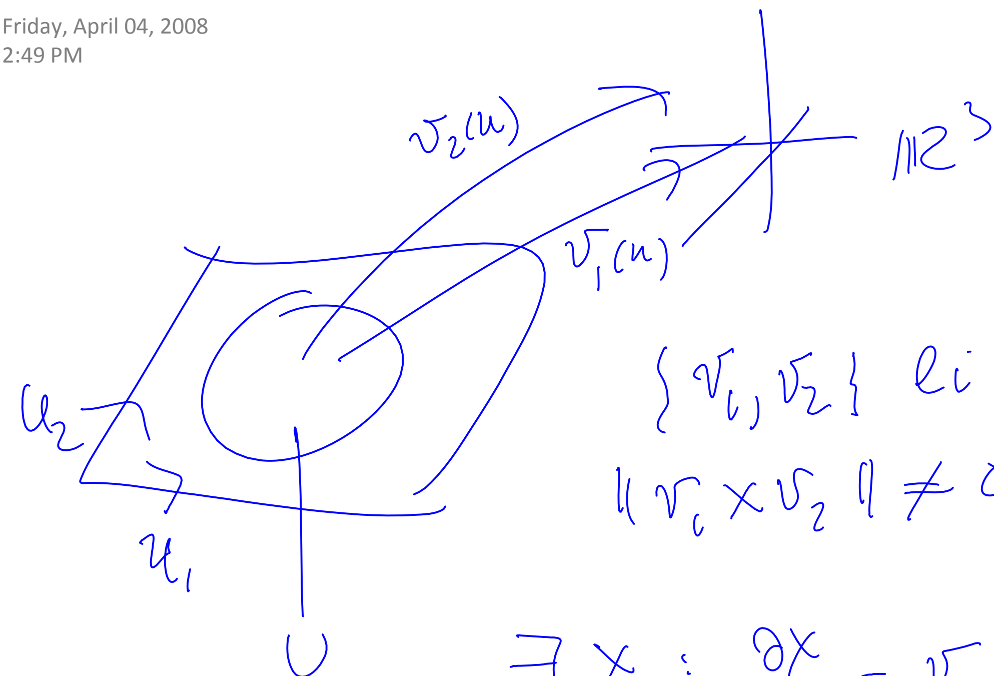
2:49 PM



$\{v_1, v_2\}$ l.i.

$\|v_1 \times v_2\| \neq 0$

$\exists X : \frac{\partial X}{\partial u_1} = v_1 \quad \partial X$