Surface Representations
Volumetric Models

ENGN2911I
3D Photography and Geometry Processing
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Implicit surfaces

- Set of zeros of a function
  - { (x,y,z) : f(x,y,z) = 0 }
- Good for boolean operations (CSG)
- Difficult to render (ray-tracing)
- Iso-surface
  - Function defined by piecewise function
  - Volumetric mesh
  - 1 function value per vertex
- Iso-surface algorithm
  - Conversion to triangle or polygon mesh representation

Implicit surfaces

- Can be used to represent the probability that a point belongs to a surface
  - Occupancy grid
- Can be used to integrate multiple measurements
- Can be used to merge multiple 3D scans

Piecewise Linear Functions

- Triangle : Barycentric coordinates
  - Triangle / Tetrahedron / Simplex
- Every point in 3D can be written as a unique affine combination of 4 non-coplanar points (affine basis)
- Every linear function in 3D can be specified by its values at the 4 vertices of an affine basis
- A piecewise-linear function is specified in 3D by its values at the vertices of a tetrahedral mesh (volumetric).

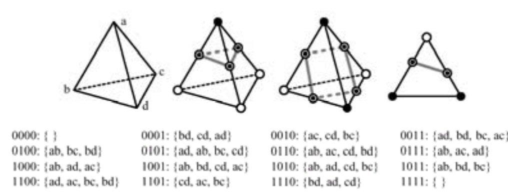
Affine bases / Linear function

$$p = \lambda_0 \; p_0 + \lambda_1 \; p_1 + \lambda_2 \; p_2 + \lambda_3 \; p_3$$

$$\begin{bmatrix} \lambda_0 \\ \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = \begin{bmatrix} p_0 & p_1 & p_2 & p_3 \\ 1 & 1 & 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} p \\ 1 \end{bmatrix}$$

$$f(p) = \lambda_0 \; f(p_0) + \lambda_1 \; f(p_1) + \lambda_2 \; f(p_2) + \lambda_3 \; f(p_3)$$

Implicit Linear Surfaces / Curves



Iso-surfaces on tetrahedral meshes

- Piecewise linear function defined on vertices of tetrahedral mesh f(i)
- For each edge (i,j) such that f(i)f(j)<0
  - create a surface vertex v(i,j)
- For each tetrahedron (i,j,k,l)
  - Skip if all vertices are positive or negative
  - Else if 3 positive or 3 negative create a triangle
  - Else (if 2 positive and 2 negative) create two trianges
- Output triangle mesh is IndexedFaceSet
- Is it a manifold mesh ? Why ?

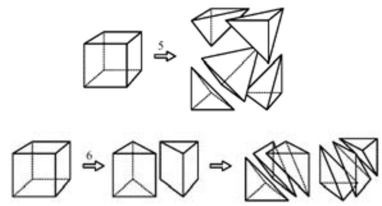


Figure 24. Five- and Six-Decompositions of the Cube

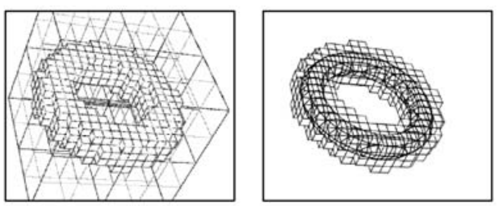
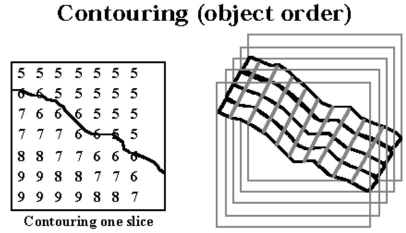


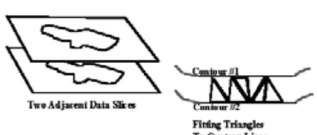
Figure 4. Subdivision Enclosing a Torus

Contouring (object order)

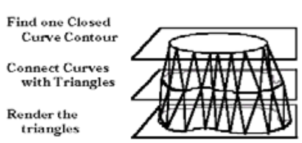


- Assumes Volume Contains Thin Boundary Surfaces
- Classify All Cells as Inside, Outside, or "On" The Surface
- Fit Constant-value Surfaces to All "On" Cells
- Render Surfaces

Connecting Slices



Connecting Slices (cont.)



Iso-surfaces on hexahedral meshes

- Function defined on vertices of regular grid
- For each edge (i,j) such that f(i)f(j)<0
  - create a surface vertex v(i,j)
- For each intersecting cube
  - Polygonize intersection
- Output triangle mesh is IndexedFaceSet
- Is it a manifold mesh ? Why ?
- Main problem: storage
- Solution: do not represent the mesh explicitly

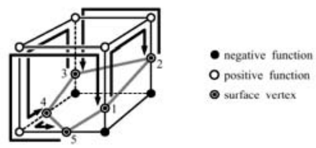
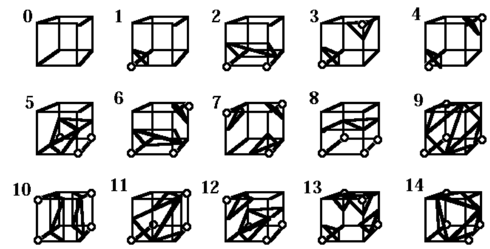


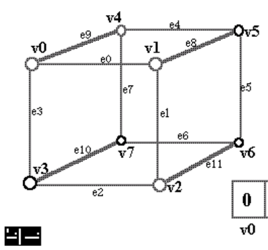
Figure 17. Cubical Polygonization

Edge Insertion Table

256 Cases Reduce to 15



Constructing an Index into the Edge Intersection Table



0	0	0	1	0	1	1	1
v0	v1	v2	v3	v4	v5	v6	v7

Marching Cubes Algorithm

1. User Specifies Threshold Value
2. Read Four Slices Into Memory
3. Scan Middle Two Slices and Create a Cell
4. Classify Eight Vertices. Construct Index Number.
5. Use Index to Look Up List of Edges
6. Find 3 Surf/Edge Intersections via Linear Interpolation
7. Calculate Unit Normal (Gradient) at 3 Intersections
8. Output the Triangle Vertices and Vertex Normals

Interpolation

- Linear interpolation
- Triangle : Barycentric coordinates
  - Triangle
  - Tetrahedron
- Quadrilateral ?
  - Bi-linear interpolation
- Cube ?
  - Tri-linear interpolation

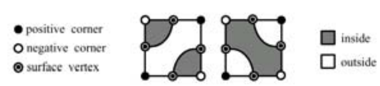


Figure 21. Alternate Surface Vertex Connections for a Face

us arrangements of ambiguous faces within a cube are shown below.

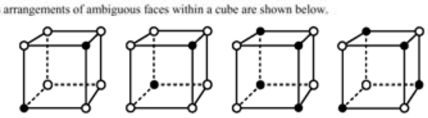


Figure 22. Ambiguous Corner Configurations for a Cube

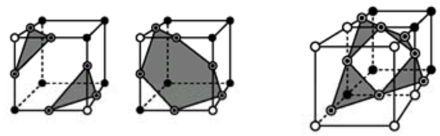


Figure 23. Inconsistent Disambiguation

Extensions

- Iso-surface algorithm assumes smooth surface without singularities
- How to represent ridges ?
- Iso-surface algorithm produces regular face sizes even in regions where fewer faces would produce equally good approximation
- Adaptive iso-surfaces ?