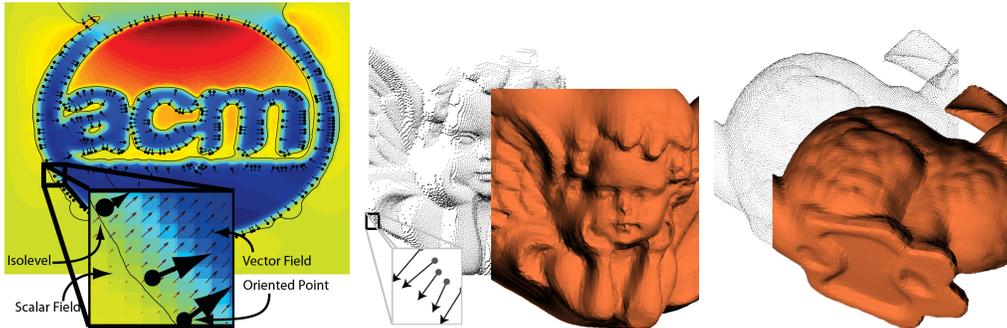


Vectorfield Isosurface-based Reconstruction from Oriented Points

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We present a new and much simpler formulation for the problem of reconstructing an implicit surface from an oriented point cloud acquired by a range scanner or a stereo vision system. Data vectors are first extended to a continuous vector field on a bounding volume, which is then integrated in the least squares sense yielding an implicit function whose zero level set approximates the data points. Function discretizations associated with regular grids automatically produce Iso-surface polygon meshes. Extrapolating missing and noisy data, integrating multiple scans, developing data structures and algorithms optimized for fast visualization and geometry processing, are challenging problems and active areas of research addressed by this work. We plan to use multi-resolution data structures to integrate streams of point clouds in real time.

Implicit representations have the advantage of dealing with arbitrary topology. [Ohtake et al. 2003] introduces an adaptive hierarchical implicit representation composed of local quadric patches and weights associated with nodes in a oct-tree. Given that for rendering or post-processing we extract an isosurface over a regular grid (e.g., via Marching Cubes), it is worth exploring reconstruction algorithms that use implicit functions defined as a regular scalar field.

In the area of geometry processing, the notion of decoupling the filtering of normal fields and geometry has emerged as a powerful method for denoising [Tasdizen et al. 2003]. We argue that a similar decoupling for the surface reconstruction problem is worth exploring. This preliminary work presents a volumetric method for surface reconstruction that directly incorporates both point and normal information. Instead of imposing constraints and regularization directly on the values of the potential (scalar) field, we impose constraints and regularization on the gradient field. We implement this using a combination of least-squares fitting and solving a Poisson problem over a uniform grid.

The general problem of implicit surface reconstruction is as follows. Given an oriented point cloud i.e., m points and their normals, $\mathcal{D} = \{(\mathbf{p}_i, \mathbf{n}_i)\}$ sampled from a surface M , compute an implicit surface $M' = \{\mathbf{p} | f(\mathbf{p}) = 0\}$ where $f : \mathbb{R}^3 \rightarrow \mathbb{R}$ and

$$\forall (\mathbf{p}_i, \mathbf{n}_i) \in \mathcal{D} \quad \nabla f(\mathbf{p}_i) = \mathbf{n}_i \text{ and } f(\mathbf{p}_i) = 0. \quad (1)$$

The least squares solution f using interpolatory constraints (1) will not, in general, produce satisfactory results without some regularization.

We represent the scalar field f as a linear combination of basis functions (e.g., trilinear) defined on a uniform Cartesian grid,

$f(\mathbf{p}) = \sum_{\alpha} f_{\alpha} \phi_{\alpha}(\mathbf{p})$. Let \mathbf{p}_{α} denote position of a grid node α . We minimize:

$$E = \sum \|v(\mathbf{p}_i) - \mathbf{n}_i\|^2 + \lambda \sum_{(\alpha, \beta)} \|v(\mathbf{p}_{\alpha}) - v(\mathbf{p}_{\beta})\|^2 \quad (2)$$

where $v : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ is a vector field, (α, β) are edges of the lattice. The second term forces the field to be smoother as λ increases. There are two approaches to solve (2). The first is to directly solve for the implicit function f set $v = \nabla f$. After solving the sparse least squares problem for f , we solve the 1D least squares problem for the isosurface s by minimizing $\sum (f(\mathbf{p}_i) - s)^2$, the solution s is the mean of f evaluated for all $\mathbf{p}_i \in \mathcal{D}$. The second approach is to (i) construct an extension \bar{n} to \mathbb{R}^3 of the sampled normal field of M , (ii) let $v = \bar{n}$ and solve (2), and (iii) compute the potential function f that minimizes $\int_{\mathbb{R}^3} \|\nabla f - \bar{n}\|^2$. The solution f is the solution of the Poisson problem $\Delta f = \nabla \cdot \bar{n}$. We represent \bar{n} using basis functions over a grid, as f above. We have experimented with the following: wavefront propagation, diffusion, and least squares fitting to the data and penalizing the curl of \bar{n} (ideally \bar{n} is integrable). We use conjugate gradient descent to solve the Poisson problem.

For both of the reconstructions shown we used a 110^3 grid with $\lambda = 0.5$ and $v = \nabla f$ in (2). The angel range image with 24K oriented points (Top Middle) took 12m17s, the sparse linear least squares solver (LSQR) converged in 246 iterations. The extracted mesh has 50K faces. Notice that sparse areas of the range image are filled in smoothly. The second set of images shows Stanford bunny with 35K oriented points, (Top Right) and reconstructed mesh with 40K faces. This took 8m04s. Notice that holes in the feet are filled. Larger point sets (e.g., Ram's head, 678K) at the same resolution grid required 11–15 minutes. Reported times don't include execution of marching cubes over the volume. The reconstructions were performed on a PC with 2.8Ghz P4 and 2GB ram.

In the future, we plan to vary λ in (2) based on sampling density. We will optimize sparse linear solver using multi-grid methods, experiment with decimation techniques, and develop more efficient storage and manipulation methods for the volumetric datasets. Then we'll apply the described methods to the problem of integrating multiple oriented point datasets on-line for partial surface reconstruction.

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TASDIZEN, T., WHITAKER, R., BURCHARD, P., AND OSHER, S. 2003. Geometric surface processing via normal maps. *ACM Trans. Graph.* 22, 4, 1012–1033.

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