Reconstructing a 3D Line from a Single Catadioptric Image

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Overview

We demonstrate that, for axial non-central optical systems, the equation of a 3D line can be estimated using only four points extracted from a single image of the line. This approach follows from a classic result in enumerative geometry: there are exactly two lines in three dimensions which intersect four lines in general position. We describe a simple algorithm to reconstruct the equation of a 3D line, evaluate the conditions for which the reconstruction fails, and present preliminary experimental results using a spherical catadioptric camera.


Typical images acquired with our axial non-central camera [2].

Catadioptric Ray Equations

To test our proposed reconstruction algorithm, we designed a novel catadioptric imaging system consisting of an array of spherical mirrors and a single, high-resolution perspective camera [2]. To analyze the process of image formation in this system, we consider rays traveling in the reverse direction (from the camera towards the mirror), as illustrated below. Note that the reflected ray R_u can be written in parametric form as

$$R_u = \{p = q(u) + \lambda v(u)\}$$

where $q(u)$ is the point of intersection of the ray u with the mirror surface and $v(u)$ is the direction of the reflected ray.
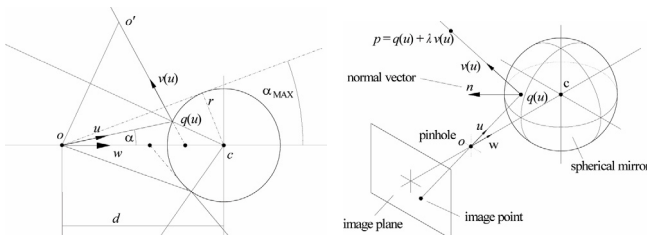

Geometric optics for an axial non-central camera. Note that the incident ray, normal vector, and reflected ray are coplanar.
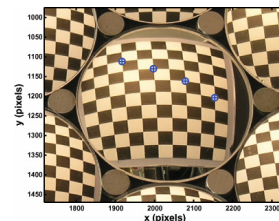
3D Line Reconstruction

Reconstruction involves: 1) calibrating the axial camera, 2) selecting four points in the image of the line, 3) determining the implicit equation for each corresponding reflected ray R_u, 4) finding the Plücker coordinates of the 3D line, and 5) obtaining an implicit representation by inverting the Plücker coordinates.

1) Intrinsic and extrinsic calibration is achieved using the method outlined by Lanman et al. [2]. (This includes determining the position and radius of the spherical mirror used in the catadioptric configuration.)

2) Four points are manually selected in the image of the 3D line. (Note that future work will focus on developing automatic methods for identifying the projection of collinear points.)

3) We determine an implicit representation for the set of four lines defined by the reflected rays R_u (corresponding to each selected image point).

4) Applying a method similar to Teller et al. [4], we find the Plücker coordinates for the two lines which intersect the set of four lines found in the previous step. (Since one solution will always be the line joining the pinhole and the center of the mirror, we obtain a unique solution.)

5) The last step is to invert the Plücker coordinates of the solution to obtain an implicit representation for the 3D line as the intersection of two orthogonal planes (one of which contains the pinhole).
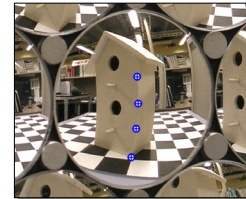
Experimental Results

Typical reconstruction results are shown below. Since ground truth is not available, we compare the single mirror reconstruction (shown in red) to that obtained using 31 mirrors (shown in green). Note that the accuracy of our reconstruction is currently limited by the calibration and the physical construction of the mirror array.


Single image reconstruction results. Left: Features used for reconstruction (crosshairs). Right: Estimated 3D line (shown in red) and viewing rays.
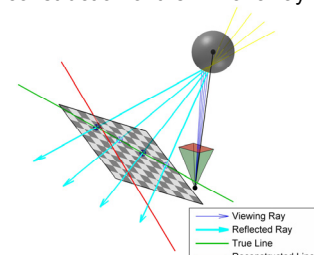
Simulation Results

In order to determine the calibration requirements, we simulated the effect of random perturbations of the reflected rays (shown in cyan) on the reconstruction. As illustrated below, we added random perturbations (distributed on spheres of radius 0.4 mm and 4 μm) to the true 3D line (shown in green). Afterwards, we estimated the equation of the intersecting line (shown in red).

Perturbation radius = 0.4 mm   *Perturbation radius = 4 μm*
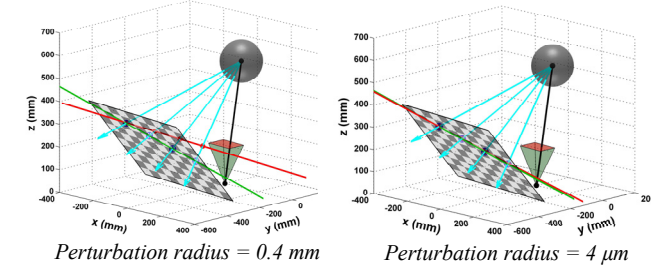
Reconstruction with Two Mirrors

We validate the experimental configuration by reconstructing a 3D line using two mirrors [3]. We begin by selecting four collinear features in each image. Next, we determine the 3D position of each feature by intersecting pairs of corresponding rays. Afterwards, a line is fit to these points using linear least-squares.

Image correspondences   *Reconstruction results (with two mirrors)*
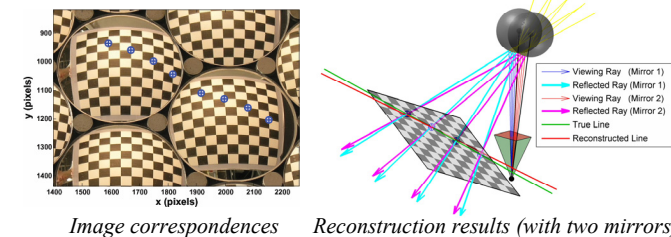
References

[1] Caglioti, V. and Gasparini, S. 2005. Localization of 3D lines from single images using off-axis catadioptric cameras. In *OMNIVIS*.

[2] Lanman, D., Crispell, D., Wachs, M., and Taubin, G. 2006. Spherical catadioptric arrays: Construction, multi-view geometry, and calibration. In *3DPVT*.

[3] Nayar, S. 1988. Sphereo: Determining depth using two specular spheres and a single camera. In *Proc. of SPIE Conference on Optics, Illumination, and Image Sensing for Machine Vision III*, 245–254.

[4] Teller, S. and Hohmeyer, M. 1999. Determining the lines through four lines. In *Journal of Graphics Tools*, 4(3):11–22.