Problem 1
Let us consider a DT LTI system $S_1$ with input $x[n]$ and output $y[n]$ related by the following equation

$$y[n] = x[n] - 4x[n - 1] + 5x[n - 1] - 2x[n - 3].$$

1. Does this equation completely determine a unique output signal for each input signal? Justify your answer. If not, what else is needed?

2. Compute the impulse response of this system.

3. Write the following input signals as linear combinations of delayed impulses

$$x_1[n] = \begin{cases} 1 : n = -1 \\ 2 : n = 0 \\ 1 : n = 1 \\ 0 : \text{otherwise} \end{cases} \quad x_2[n] = \begin{cases} 1 : n = -2 \\ 5 : n = 0 \\ 1 : n = 7 \\ 0 : \text{otherwise} \end{cases}$$

4. Compute the corresponding output signals $y_1[n]$ and $y_2[n].$

5. Compute the output signals $y_3[n]$ and $y_4[n]$ corresponding to the input signals $x_3[n] = 2^n$ and $x_4[n] = \delta.$

6. Is this system invertible?

7. Find a third signal $x_5[n],$ linearly independent from $x_3[n]$ and $x_4[n],$ which produces zero output $y_5[n] = 0.$ Are there any more linearly independent signals which produce zero output?

Problem 2
Let us consider a DT LTI system $S_2$ with input $x[n]$ and output $y[n]$ related by the following equation (note that the roles of $x$ and $y$ have been switched with respect to the system in problem 1).

$$x[n] = y[n] - 4y[n - 1] + 5y[n - 1] - 2y[n - 3].$$

1. Does this equation completely determine a unique output signal for each input signal? Justify your answer. If not, what else is needed?

2. Compute the impulse response of this system.

3. Compute the output signals $y_1[n]$ and $y_2[n]$ corresponding to the input signals $x_1[n]$ and $x_2[n]$ of problem 1. Are these signals of finite duration?

4. Is this system invertible?

5. Write the forward recursive equation for this system, and draw a system diagram interconnecting one-step delay blocks $D (x[n] \rightarrow y[n] = x[n - 1]).$

6. Write the backward recursive equation for the system and compute the output terms $y[-10], y[-9], \ldots, y[-1]$ for the input $x[n] = u[-n],$ under the additional constraint $y[n] = 0$ for $n \geq 0.$

Problem 3
OW 3.48 b), c), e) and h)

Problem 4
OW 3.50

Problem 5
OW 3.58

Problem 6
OW 3.60 b), c), e) and h)