Implicit surfaces

- Set of zeros of a function
- \( \{(x,y,z) : f(x,y,z) = 0\} \)
- Good for boolean operations (CSG)
- Difficult to render (ray-tracing)
- Iso-surface
  - Function defined by piecewise function
  - Volumetric mesh
  - 1 function value per vertex
- Iso-surface algorithm
  - Conversion to triangle or polygon mesh representation

Implicit Linear Surfaces / Curves

- \( f(p) = \lambda_0 f(p_0) + \lambda_1 f(p_1) + \lambda_2 f(p_2) + \lambda_3 f(p_3) \)

Implicit Linear Surfaces / Curves

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Affine bases / Linear function

\[
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\]

\[
\begin{bmatrix}
\lambda_0 \\
\lambda_1 \\
\lambda_2 \\
\lambda_3
\end{bmatrix}
= \begin{bmatrix}
p_0 & p_1 & p_2 & p_3
\end{bmatrix}^{-1}
\begin{bmatrix}
p
\end{bmatrix}
\]

Piecewise Linear Functions

- Triangle: Barycentric coordinates
- Triangle / Tetrahedron / Simplex

- Every point in 3D can be written as a unique affine combination of 4 non-coplanar points (affine basis)

- Every linear function in 3D can be specified by its values at the 4 vertices of an affine basis

- A piecewise-linear function is specified in 3D by its values at the vertices of a tetrahedral mesh (volumetric).

Surface Representations

- Volumetric Models

EN-193s08 3D Photography
Brown Fall 2003
Gabriel Taubin

Implicit surfaces

- Can be used to represent the probability that a point belongs to a surface
  - Occupancy grid
- Can be used to integrate multiple measurements
- Can be used to merge multiple 3D scans

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Iso-surfaces on tetrahedral meshes
- Piecewise linear function defined on vertices of tetrahedral mesh \( f(i) \)
- For each edge \((i,j)\) such that \( f(i)f(j)<0 \)
  - create a surface vertex \( v(i,j) \)
- For each tetrahedron \((i,j,k,l)\)
  - Skip if all vertices are positive or negative
  - Else if 3 positive or 3 negative create a triangle
  - Else (if 2 positive and 2 negative) create two triangles
- Output triangle mesh is IndexedFaceSet
- Is it a manifold mesh? Why?

Iso-surfaces on hexahedral meshes
- Function defined on vertices of regular grid
- For each edge \((i,j)\) such that \( f(i)f(j)>0 \)
  - create a surface vertex \( v(i,j) \)
- For each intersecting cube
  - Polygonize intersection
- Output triangle mesh is IndexedFaceSet
- Is it a manifold mesh? Why?
- Main problem: storage
- Solution: do not represent the mesh explicitly
Interpolation

- Linear interpolation
- Triangle: Barycentric coordinates
  - Triangle
  - Tetrahedron
- Quadrilateral?
  - Bi-linear interpolation
- Cube?
  - Tri-linear interpolation
Extensions

- Iso-surface algorithm assumes smooth surface without singularities
- How to represent ridges?
- Iso-surface algorithm produces regular face sizes even in regions where fewer faces would produce equally good approximation
- Adaptive iso-surfaces?