Homework I

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1 3D transformations

Exercise 1. Prove Rodrigues' formula: for every unit-length vector r and angle α, the matrix of equation 1 (see *rotations* handout) is a rotation matrix that corresponds to a rotation of α radians around the vector r. (hint: first consider the case $r = (0,0,1)^t$; for the general case consider two other unit-length vectors u and v that jointly with r define an orthogonal matrix $Q = [uvr]$, and study the matrix QRQ^t).

Exercise 2. Implement Rodrigues' formula. Validate the routine by computing the rotation matrix R that encodes a rotation by $\theta = 30$ degree around the vector $n = (1,1,1)^t$.

Exercise 3. Implement the inverse of Rodrigues' formula. Validate the routine by verifying that, given the R from ex.2 , the output is again $r = n/\|n\|$ and $\theta = 30$.

Exercise 4. Rodrigues' formula defines a function

$$\{\omega : 0 < |\omega| \leq 1\} \rightarrow \mathrm{SO}(3) \ ,$$

with $s = (|\omega|^2)^{1/2}$, $c = (1 - |\omega|^2)^{1/2}$, and $r = \omega/s$, which is clearly $1-1$ and continuous. Show that the function is well defined and continuous at $\omega = 0$.

Exercise 5. What rotations matrices correspond to $|\omega| = 1$?. What angle of rotation these matrices correspond to ?

2 Triangulation

Exercise 6. Implement the triangulation code in `StereoTriangulator.java` and test it on the five data sets provided. Save the reconstructed 3D objects as VRML files. To show the reconstructed geometry, choose different viewpoints and save the corresponding images.

3 Error Analysis

Exercise 7. In this exercise we want to verify that the reconstruction error of the triangulation procedure is function of the baseline. Start your Java application and load the "bun000" left-right image pair. Then, triangulate the image pair selecting 5 different baselines (e.g. start from the default one and then use the "double baseline" button to increase it). After each triangulation, save the 3D coordinates by using the "save 3D coordinates button". A bun000.3dp file is generated each time. We provide a file (bun000real.3dp) with the real 3D object coordinates of the bunny. Plot the reconstruction error E as function of the baseline. The reconstruction error may be expresses as follows:

$$E = \sum_{n=1}^{N} (X_n^e - X_n^r)^2 \qquad (1)$$

where X_n^r is the 3D coordinate vector of the n^{th} reconstructed point and X_n^r is the 3D coordinate vector of the n^{th} real point. N is the total number of 3D points. Comment the results.

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