Objective:
To create a Phase Shifting (PS) method for 3D scanning robust to global illumination effects and with low decoding computational cost.

Related work:
- Multiple PS [1] projects patterns of multiple frequencies and uses Temporal Phase Unwrapping [2], which is accurate and computationally efficient, but it is not robust to global illumination effects.
- Micro PS [3] is robust to global illumination effects, because it uses only high frequency patterns, but decoding requires search in a LUT which is inefficient and produces frequent errors.

Embedded PS:
- High frequency patterns robust to global illumination
- Embedded low frequencies permit efficient and accurate phase unwrapping
- Every projected frequency generates a depth measurement, multiple measurements combined produce more accurate results.

Coding Method:
Let be \( \{ T_1, \ldots, T_M \} \) real numbers greater than 1. We define M embedded frequencies \( F_m \) as:
\[
F_m = \frac{1}{T_m} \cdots \frac{1}{T_m}.
\]
We define M pattern frequencies \( f_m \) as:
\[
\begin{bmatrix}
    f_1 \\
    f_2 \\
    f_3 \\
    \vdots \\
    f_M
\end{bmatrix} = \begin{bmatrix}
    1 & 0 & 0 & \cdots & 0 \\
    1 & 1 & 0 & \cdots & 0 \\
    1 & 0 & 1 & \cdots & 0 \\
    \vdots & \vdots & \vdots & \ddots & \vdots \\
    1 & 0 & 0 & \cdots & 1
\end{bmatrix} \begin{bmatrix}
    F_1 \\
    F_2 \\
    F_3 \\
    \vdots \\
    F_M
\end{bmatrix}.
\]
Create a sinusoidal pattern for each of the M pattern frequencies.

Example: if \( T_1=16, T_2=8, \) and \( T_3=8 \); then
- embedded frequencies are \( F_1=1/16, F_2=1/128, \) and \( F_3=1/1024 \)
- pattern frequencies are \( f_1=1/16, f_2=1/14.22, \) and \( f_3=1/15.75 \)

Decoding Method:
Use image intensities to recover the phase vector:
\[
u = [0, c_1, s_1, \ldots, c_M, s_M]^T, \quad c_m = a_m \cos(\omega_m \phi_m), \quad s_m = a_m \sin(\omega_m \phi_m).
\]
Each relative phase \( \phi_m \) corresponds to a pattern frequency. Extract the embedded frequencies:
\[
\phi_1 \equiv \phi_1, \quad \phi_m = \phi_m - \phi_1 \quad \text{for} \ m > 1.
\]
Use Temporal Phase Unwrapping [2] to unwrap the embedded frequency phases \( \phi_m \) and pattern frequency phases \( \phi_m \).
Each pattern frequency phase provides a separate depth measurement.

Comparison of PS unwrapping methods:
- **Profile of a metal plane:**
  - Micro PS produces frequent unwrapping errors, some of them can be corrected with a median filter.
  - Multiple PS and Embedded PS show now errors.

Surface Properties:
- **a) Lambertian Surface:** in the absence of global illumination effects Embedded PS and Multiple PS performs similarly.
- **b) Specular Concave Surface:** Embedded PS and Micro PS performs well in the presence of global illumination effects

Multiple Measurements:
3D models from same image data using only a single measurement (blue) and multiple measurements (red). The latter one has less noise and it is more accurate.

3D Model Comparison:
- **Embedded PS:** Bowl, 15 images
- **Micro PS:** Rectangular Bowl: concave object with specular surface, exhibit strong interreflections. Leaf, 7 images
- **Multiple PS:** Tree Leaf: fine structures with small details. Orange, 6 images

Computation Complexity:
Plot of the decoding algorithm computation time vs. the number of captured images.

Conclusion:
Embedded PS is robust to global illumination effects, it is fast and accurate, and produces multiple measurements which are used to generate 3D models with very low noise.

References: